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Probabilistic presentation of the total bending moments of FPSO's

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ABSTRACT: The wide spread practice for presentation of the still water loads acting on the hull structure of FPSO's is to use the individual amplitude statistics. When the probability density function of the loads is built, its integration provides the probability of exceeding any given level of loads. Also, once it is available, one can apply the principles of extreme value statistics to obtain the highest value in any number of cycles. The individual amplitude statistics is used in fatigue strength calculations while the extreme value statistics can be used in ultimate strength calculations. An example is given for the probability density functions of a sample FPSO when the two approaches are applied for assessment of the total hull girder load that can be applied for calculation of its ultimate strength.

1 INRODUCTION

Traditionally, in shipbuilding, the total shear forces and bending moments are calculated as a sum of the still water and wave-induced loads (see, e.g., Hughes O F, 1983). This practice has been in existence for more than a century. It has been introduced for convenience in the calculations because the variability of these two processes is different. In reality, the load acting on the hull is the total load but its calculation is difficult because of the uncertainties in predicting the sea environment and loading patterns the ship will sustain over its life. The uncertainty related to wave-induced loads is greater than that for still water loads. As to the still water loads, the presumption is that they can be controlled and more accurately predicted, especially now with the availability of onboard computers.

The situation with FPSOs is different from that for ships. In most cases, they are operating in benign sea environment. In addition, even if they are located in harsh sea environment, they work as stationary structures. Thus, prediction of the sea loads for FPSOs is more accurate due to the existence of comprehensive data for the world oceans and seas (see, e.g., OCEANOR Oceanographic Company). Again, the final goal is to predict the total load acting on the FPSO hull structure. It requires statistical analysis of the total shear forces and bending moments for each specific FPSO type and sea environment, which includes statistical analysis of its two components. In this sense, collection of statistical data for still water loads and their analysis is still a worthy activity.

With this in mind, the paper presents the results of statistical analysis of still water and wave-induced bending moments of an FPSO using individual amplitude statistics (i.e., statistical analysis of all individual amplitudes) and extreme value statistics, which can be used in predicting the probabilistic distribution of the total bending moments.

2 RAW STATISTICAL DATA FOR THE STILL WATER BENDING MOMENTS

Records of the still water bending moments (M_{SW}) within the parallel middle body of a FPSO for dozens of months from 2003 till 2007 have been analyzed and shown in Fig. 1 - Fig. 7. In general, the records were made once a day (in the afternoon) although some days no records were made (this explains the non-equidistant points in the graphs). The sign change of M_{SW} is due to the change of the loading pattern at the time of recording.

One should note that the recorded M_{SW} were not at the same sections although all of them were within the region of $0.4L_{BP}$. However, for the statistical analysis, the data were mixed on the ground that within the parallel middle body (or within the region of $0.4L_{BP}$) the hull structure scantlings are the same. For slow-going ships such as tankers (the FPSO was refurbished from a tanker), this region may be even longer. Therefore, the location of the recorded M_{SW} within this region is not very important because the calculated bending stresses within the region will be the same.

3 PROBABILISTIC DISTRIBUTION OF THE BENDING MOMENTS DERIVED BY THE INDIVIDUAL AMPLITUDE STATISTICS

3.1 *Still Water Loads*

The history of probabilistic presentation of still water loads is given in the paper of Ivanov L D and Wang Ge , 2008. In this paper, goodness of fit has been performed by the computer program EasyFit (see Mathwave Data Analysis & Simulation Company) to analyze the raw statistical data. Two criteria have been used – those of Kolmogorov-Smirnov and Andersen-Darling. The results do not always coincide but, in general, they are close to each other. Following the usual practice in shipbuilding, the hogging and sagging M_{SW} are analyzed separately. Preference is given to probabilistic distributions that are most commonly used, such as Gaussian, Rayleigh, Weibull, Log-normal (see Fig. 8 and Fig. 9). The notation “R-i” denotes the ranking of the corresponding theoretical probabilistic distribution when Kolmogorov-Smirnov criterion is used.

The shape parameter of Weibull distribution in Fig. 8 is ≈ 1 and this is the reason for the Weibull distribution to almost coincide with the exponential distribution. One can notice in Fig. 8 and Fig. 9 that probabilistic distributions that fit hogging M_{SW} may not fit the sagging M_{SW}. For this reason only the Gaussian distribution is shown in Fig. 9 because the other distributions shown in Fig. 8 are rejected by the Kolmogorov-Smirnov criterion. The derived probabilistic distributions should be truncated because the M_{SW} cannot decrease/increase indefinitely. The lower boundary should not be zero. It is possible that at a section within the parallel middle body (or the region of $0.4L_{BP}$) the M_{SW} is equal to zero. However, we are interested in the maximum M_{SW} within the region. It is not possible that at each section the M_{SW} will be equal to zero (it would be zero only in the case when the distribution laws of the buoyancy and gravity forces within this region are exactly the same, which cannot happen in real ships). In the paper, the numerical value of the lower boundary is determined based on the available records for the M_{SW} (it is equal to 6% of the design M_{SW} which is ± 10780 MN.m). The upper boundary is the permissible M_{SW} in the Rules (on the premise that the operators strictly follow the loading manual for operation of the FPSO). Mark the boundaries of the M_{SW} as follows:

b_u = maximum possible value of the M_{SW}
b_l = minimum possible value of the M_{SW}

In the paper, Gaussian distribution of the M_{SW} is recommended. For this case, one can find the equation of the truncated normal distribution based on the premise that the area below the truncated normal distribution with boundaries b_u , b_l should be equal to unity (as for a normal distribution with boundaries - ∞, + ∞). The difference between the ordinates of the two probability density functions (p.d.f.) will be a constant C_a , which can be determined in the following way:

$$\int_{b_l}^{b_u} f_c(y)\,dy = C_a\left[F(b_u)-F(b_l)\right]=1 \qquad (1)$$

$$C_a = \frac{1}{F(b_u)-F(b_l)} \qquad (2)$$

$F(b_u)$, $F(b_l)$ = cumulative distribution function (c.d.f.), correspondingly, for $y = b_u$ and $y = b_l$, i.e.

$$F(b_u)=\Phi\left(\frac{b_u-\bar{y}}{\sigma_y}\right) \qquad F(b_l)=\Phi\left(\frac{b_l-\bar{y}}{\sigma_y}\right) \qquad (3)$$

Thus, the truncated normal distribution function of the M_{SW}, $f_c(y)$, will be:

$$f_c(y)=\frac{1}{\Phi\left(\frac{b_u-\bar{y}}{\sigma_y}\right)-\Phi\left(\frac{b_l-\bar{y}}{\sigma_y}\right)}f(y) \qquad (4)$$

$$f(y)=\frac{1}{\sigma_y\sqrt{2\pi}}\exp\left[-\left(\frac{y-\bar{y}}{\sigma_y\sqrt{2}}\right)^2\right] \qquad (5)$$

where $y = M_{SW}$, $f(y)$ = Gaussian p.d.f. of M_{SW}
$\bar{y}$ = mean value of "y", σ_y = standard deviation of "y", Φ = Laplace integral of probability

$$\Phi\left(\frac{y-\bar{y}}{\sigma_y}\right)=\frac{2}{\sqrt{2\pi}}\int_0^{\frac{y-\bar{y}}{\sigma_y}}\exp\left[-\left(\frac{y-\bar{y}}{\sigma_y\sqrt{2}}\right)^2\right]dy \qquad (6)$$

The truncated Gaussian distributions treated separately for hogging and sagging M_{SW} are shown in Fig. 10. In principle, there is only one probabilistic distribution for the M_{SW}. Its separate treatment for hogging and sagging is performed for convenience in the calculations. The area below each truncated Gaussian p.d.f. (for hogging and sagging) in Fig. 10 is equal to unity because the two cases are treated separately.

However, the phenomenon (i.e., M_{SW}) is one and should have one probabilistic distribution but with two modes. Therefore, the following method is used to build the bi-modal p.d.f. for hogging and sagging. The major assumptions are:

- The sum of the areas below the two p.d.f. is equal to unity
- The ratio between the two areas below the p.d.f. is equal to the ratio between the number of cases with hogging M_{SW} and sagging M_{SW}.

In mathematical terms, these assumptions can be presented in the following way:

$$\left| \begin{array}{l} K_s \int_{b_l}^{b_u} f_s(y)\,dy + K_h \int_{b_l}^{b_u} f_h(y)\,dy = 1 \\ K_h \int_{b_l}^{b_u} f_h(y)\,dy / K_s \int_{b_l}^{b_u} f_s(y)\,dy = \alpha \end{array} \right. \quad (7)$$

where :
K_s = unknown coefficient to multiply each ordinate of the p.d.f. for sagging M_{SW}
K_h = unknown coefficient to multiply each ordinate of the p.d.f. for hogging M_{SW}
α = coefficient equal to the ratio between the number of cases with hogging M_{SW}, N_h, and the number of cases with sagging M_{SW}, N_s, i.e.

$$\alpha = N_h / N_s \quad (8)$$

However, the area below each p.d.f. is equal to unity, i.e.

$$\int_{b_l}^{b_u} f_s(y)\,dy = 1 \qquad \int_{b_l}^{b_u} f_h(y)\,dy = 1 \quad (9)$$

Thus, Eq. (7) is reduced to the following system of equations:

$$\left| \begin{array}{l} K_h + K_s = 1 \\ K_h / K_s = \alpha \end{array} \right. \quad (10)$$

The solution of Eq. (10) is simple, i.e.:

$$K_s = 1/(1+\alpha) \qquad K_h = 1 - K_s \quad (11)$$

Naturally, if the total number of records of sagging and hogging M_{SW} and the separate number of records for sagging (or hogging) M_{SW} are known, the unknown coefficient will be only one. Once the coefficients K_s and K_h are known, the bi-modal p.d.f. can be calculated.

Fig. 11 and Fig. 12 show the histogram and the corresponding probability density function for hogging and sagging M_{SW} when they are considered as two sides of one phenomenon (i.e., the p.d.f. of M_{SW} is a bi-modal one). It is worth noting that the area below the two parts of the bi-modal p.d.f. is equal to unity.

As an example, the result for the FPSO under consideration is shown in Fig. 13 (for other FPSO, the upper and lower boundaries may be different but the shape of the bi-modal p.d.f. will be very similar to that on Fig. 13). Fig. 14 presents the p.o.e. in logarithmic scale, which is a convenient way to illustrate the behavior of the functions in the asymptotic tails.

It never happens that the maximal M_{SW} equals zero. This is the reason that within the range of M_{SW} = -633400 and 633400 KN.m, the ordinates of the p.d.f. are zero (for other FPSO this range might be different but a range with zero ordinates of the p.d.f. of M_{SW} will still exist).

One should be careful when using the c.d.f. in Fig. 13. By definition, each ordinate of the c.d.f. is equal to the area below the p.d.f., i.e. it represents the probability that "y" is smaller than the selected value of "y". In this case, this definition is not valid. An example of how to use the so derived probabilistic distributions is given below.

If one wants to calculate, e.g., the probability that the sagging M_{SW} will be greater than -1835 MN.m, the probability will be 0.23. Note, that the ordinate of the c.d.f. provides the probability that (in physical terms) sagging M_{SW} will be greater than the given value (not smaller, as by the traditional mathematical definition of the c.d.f.). If one wants to calculate the probability that the hogging M_{SW} will be greater than +1880 MN.m, the probability will be 1 – 0.33 = 0.67 (0.33 is the ordinate of the c.d.f. for hogging M_{SW} at M_{SW} = 1880 MN.m). Following the two examples, one can calculate the probability that the M_{SW} will be greater than any given positive or negative (i.e., hogging or sagging) M_{SW} using the c.d.f. for negative M_{SW} and the p.o.e. - for positive M_{SW}.

3.2 *Wave induced bending moment*

The wave-induced bending moment (M_W) is calculated following the ABS Rules, 2008, for hogging, $M_{w,h}$, and sagging, $M_{w,s}$, bending moment:

$$M_{wh} = 190 \beta C_1 L^2 B C_B .10^{-3} \quad \text{[KN.m]} \quad (12)$$

$$M_{ws} = -110 \beta C_1 L^2 B (C_B + 0.7).10^{-3} \quad \text{[KN.m]} \quad (13)$$

where:

$$\left. \begin{array}{ll} C_1 = 10.75 - \left(\dfrac{300-L}{100} \right)^{1.5} & \text{for } 90 \le L \le 300 \text{ m} \\ C_1 = 10.75 & \text{for } 300 \le L \le 350 \text{ m} \\ C_1 = 10.75 - \left(\dfrac{350-L}{150} \right)^{1.5} & \text{for } 350 \le L \le 500 \text{ m} \end{array} \right\} \quad (14)$$

β = environmental severity factor (in the example, it is equal to 0.67)

The probability of exceedence (p.o.e.) of the design wave-induced bending moment in the Classification Societies Rules is assumed to be 10^{-8} (it means one exceedence in one hundred million load cycles, which means approximately within around ship's service life T_o=20 years). It is also known that the probabilistic distribution is assumed to be two-parameter Weibull distribution (Ochi M, 1989), i.e.

$$f(y)=\frac{\lambda}{\alpha}\left(\frac{y}{\lambda}\right)^{\lambda-1}\exp\left[-\frac{1}{2}\left(\frac{y}{\alpha}\right)^{\lambda}\right]$$
$$F(y)=1-\exp\left[-\frac{1}{2}\left(\frac{y}{\alpha}\right)^{\lambda}\right] \quad (15)$$

where f(y) = p.d.f. , F(y) = c.d.f. , α = scale parameter, λ = shape parameter. The parameters λ and α are obtained following the method developed by Kamenov-Toshkov et al, 2006, λ = 0.8258, α = -196090 KN.m for sagging and α = 183740 KN.m for hogging.

The values of the M_W however cannot reach plus/minus infinity because the Ocean wave energy is not infinite, indicating that its probabilistic distributions should be truncated. The ordinates of the truncated probabilistic distributions are obtained in the same way as are for the truncated probabilistic distributions already done for the still water loads. For hogging and sagging M_W, the lower boundary is zero (it is possible to have calm sea) while the upper boundary could be either the design M_W or above it to consider extraordinary large wave load. In the paper, it is assumed 10% above the design M_W (the value can easily be replaced when more accurate data is available). The bi-modal p.d.f. of M_W when hogging and sagging are treated separately is shown in Fig. 15. As for still water loads, it is not possible for the ship to be exposed simultaneously to hogging and sagging wave-induced bending moment, M_W. Therefore, if the wave-induced load cycles within a given ship's life-span are taken as 100%, as a first approximation, 50% hogging and 50% sagging within the given life-span can be assumed (i.e., $K_{sag,W}$ = 0.5 and $K_{hog,W}$ = 0.5). The bi-modal p.d.f.s are calculated and shown in Fig. 16 and Fig. 17 together with the design M_W calculated with the formulae in ABS Rules, 2008. The p.o.e. of M_W is shown in logarithmic scale in Fig. 18.

3.3 *Probabilistic distribution of the total bending moment*

There are several proposals in which it is possible to combine M_{SW} and M_W (e.g., Ferry Borges, Castanheta M, 1968, 1971; Soares C G, 1992; Söding H, 1979; Turkstra C J, 1970). Any of them can be used provided the final results are calibrated against real experience from ship operation. The difficulty in calculating the p.d.f. of the total bending moment, M_t, is in the fact that there is insufficient data for the total loads acting on the ship structure during relatively long periods (e.g., two or three years). The attention of the researchers was concentrated on deriving data for the probabilistic distribution of the wave-induced loads as the key contributor to the uncertainties in loads' assessment. Less attention was paid to the combination of the still water and wave-induced loads. The general thinking was that the still water loads are controllable and the uncertainties in their calculations are smaller than those for the wave loads.

Under these circumstances, the authors calculated the probabilistic distributions of the total bending moments by the rules for the composition of the distribution laws of the constituent variables (Soares CG, 1992, Söding H, 1979, Suhir E, 1997]):

$$F_{M_t}(M_t)=\int_0^{\infty} F_{M_{SW}}(M_t\text{-}M_W)f_{M_W}\left(M_W\right)d\left(M_W\right) \quad (16)$$

where

$f_{M_t}(M_t)$= p.d.f. of the total bending moment M_t

$F_{M_t}(M_t)$= c.d.f. of M_t,

$f_{M_W}(M_W)$ = p.d.f. of M_W

$F_{M_{SW}}(M_{SW}=M_t-M_W)$= c.d.f. of M_{SW}

$f_{M_{SW}}(M_{SW})$= p.d.f. of M_{SW}

$F_{M_W}(M_W=M_t-M_{SW})$= c.d.f. of M_W

The distribution functions of M_t derived by Eq. (16) are shown in Fig. 19, the p.o.e. of M_t is shown in Fig. 20. For comparison, all p.o.e. of M_t are shown in Fig. 21.

4 ANALYSIS BY THE STATISTICS OF EXTREMES

When the ship's hull ultimate strength is to be checked, the extreme value statistics (Kotz S, Nadarajah S, 2000) for still water and wave-induced bending moments are used. In this case, this approach is more reasonable than the use of individual amplitude statistics. Type one of the probabilistic distributions of extremes is derived for hogging and sagging M_W and M_{SW} by the formula:

$$F_e\left(y_e\right)=\exp\left\{-\exp\left[-\alpha\left(y_e-\mu\right)\right]\right\} \quad (17)$$

where α and μ are parameters of the extreme value distribution type one and y_e is the corresponding random variant (i.e., M_{SW} or M_W). The parameters α and μ are calculated following the procedure described by Ochi M, 1989.

The parameter μ is the probable extreme value expected to occur in "n" observations. It can be evaluated from the initial c.d.f. of M_{SW} or M_W for which the probability of exceeding this value is 1/n, i.e.:

$$F(\mu) = 1 - 1/n \quad (18)$$

where F is the initial c.d.f. of M_W or M_{SW} calculated for μ.

For the wave-induced bending moment, this is the design wave-induced bending moment (calculated by the formulae in ABS Rules, 2008, for sagging and hogging with p.o.e. 10^{-8}).

The other parameter α is calculated by the formulae (Ochi M, 1989):

$$\alpha = \frac{f(\mu)}{1 - F(\mu)} \quad (19)$$

where f is the initial p.d.f. of M_W or M_{SW} calculated for μ.

For the wave-induced bending moments the following values for the parameters α and μ were obtained:
For sagging: μ = 6678920 [KN.m] , $\alpha = 2.28*10^{-6}$
For hogging: μ = 6258260 [KN.m] , $\alpha = 2.43*10^{-6}$

For the still water bending moments the following values for the parameters α and μ were obtained:
For sagging: μ = - 6825100 [KN.m] , $\alpha = 3.05*10^{-6}$
For hogging: μ = 7345000 [KN.m] , $\alpha = 2.05*10^{-6}$

The results of the calculations with Eq. (17) for M_{SW} and M_W are shown in Fig. 22 – Fig. 25 together with the results obtained when individual amplitude statistics are used.

All distribution functions for M_{SW} and M_W obtained by the extreme value statistics are shown in Fig. 26 and Fig. 27. The p.o.e. of the design M_{SW} and M_W, when extreme statistics and ordinary statistics are used, is given in Fig. 28 and Fig. 29 in logarithmic scale while the p.o.e. of M_{SW}, M_W and M_t are shown in Fig. 30 in logarithmic scale as well.

After all necessary parameters were calculated, a parametric study was performed to find the relationship between the probability of exceeding any given limit either presented in absolute numbers (i.e., the value of the permissible M_t for sagging and hogging) or as a function of the coefficient ϕ in the following formula:

$$\text{permissible } M(\text{total}) = \phi(\text{design } M_{SW} + \text{design } M_W) \quad (20)$$

The results of the parametric study are graphically illustrated in Fig. 31 and Fig. 32.

5 COMMENTS ON THE NUMERICAL RESULTS

When individual amplitude statistics are used, the p.o.e. of the design M_{SW} and M_W is negligible. The p.o.e. of the design wave-induced bending moment (hogging and sagging) is 10^{-8}. The p.o.e. of the design still water bending moment is around $2.2x10^{-7}$ for hogging and $1.3x10^{-11}$ for sagging.

When the design total bending moment M_t is calculated with coefficient ϕ = 0.875 (see Eq. (20)), the p.o.e. of the design total bending moment is given in Fig. 21. One should emphasize that the absolute value of the total design bending moment is given here only as an example. The procedure, however, is a general one and can be applied to any other value of the permissible total bending moment.

The calculations showed that, when extreme value statistics are applied, the p.o.e. of the design wave-induced bending moment and the design total bending moment in class rules becomes substantial.

It is a well established practice to apply the extreme value statistics for control of the hull girder ultimate strength, which depends on M_t. However, even with the application of extreme value statistics, the hull girder collapse is a very unlikely event. One of the reasons is that this FPSO operates in benign environment. Another reason is the practical experience and theoretical knowledge incorporated in classification societies' Rules, which ensure high reliability level of the hull girder. One should also bear in mind that the permissible stresses are method dependent. Hence, when extreme value statistics are applied, one should recalibrate the permissible stresses in the rules to reflect the application of new methodology.

All calculations in the paper refer to the so-called "life time" approach, i.e. all probabilistic distributions refer to specific service life of the FPSO (in the example, 20 years service life). This approach does not contradict the so-called "annual" approach when the loads, the corresponding geometric properties and the probability of failure are also calculated on the annual basis. In the former approach, an "average" reliability of the hull structure is calculated while the latter approach provides information for the year when structural failure may occur. Both approaches are needed for concise assessment of the hull structure reliability at the design and operation stage.

6 CONCLUSION

A method is proposed and tested to present the probabilistic distributions of hogging and sagging still water and wave-induced loads by bi-modal p.d.f. The sum of the areas below the two parts of the p.d.f.'s (for hogging and sagging) should be equal to unity. The area below each p.d.f can be determined from statistical data or assumed based on experience.

The parameters of the probabilistic distributions of the wave-induced loads are calculated by the method proposed by Kamenov-Toshkov et al 2006, which allows for determining these parameters for any duration of the service life. The probabilistic distribution of the total loads is calculated by the rules of the composition of the distribution laws of the constituent variables. This allows for determining the p.o.e. of any given or assumed total load.

The application of extreme value statistics for calculation of the probability that the total hull girder bending moment will exceed the design total bending moment confirmed the very high reliability of the hull girder.

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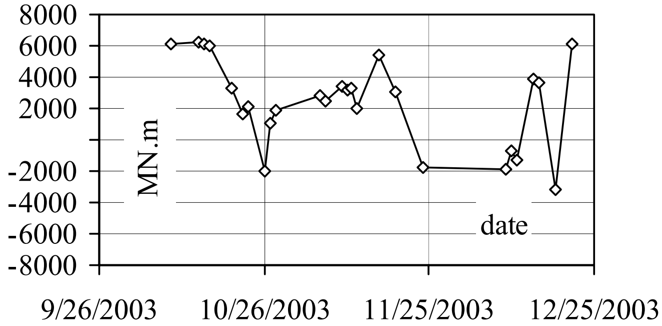

Fig. 1 M_{SW} distribution for Oct.-Dec. 2003

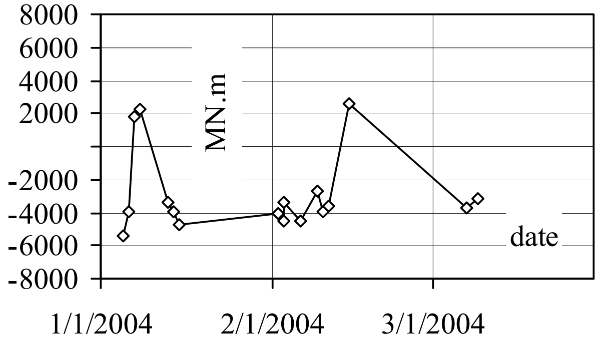

Fig. 2 M_{SW} distribution for Jan. 2004-March 2004

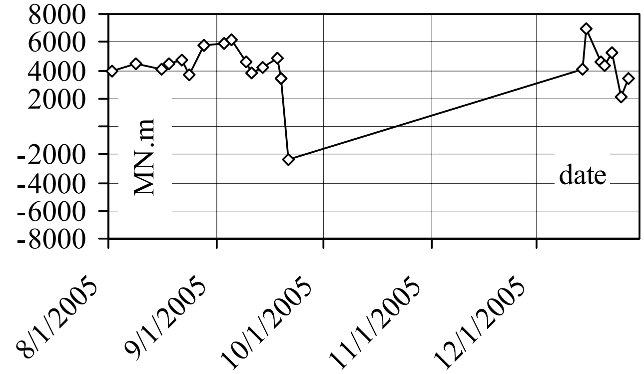

Fig. 3 M_{SW} distribution for Aug. 2005-Dec. 2005

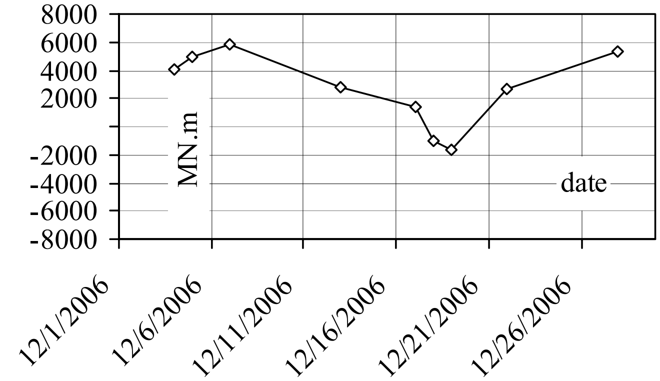

Fig. 4 M_{SW} distribution for Dec. 2006

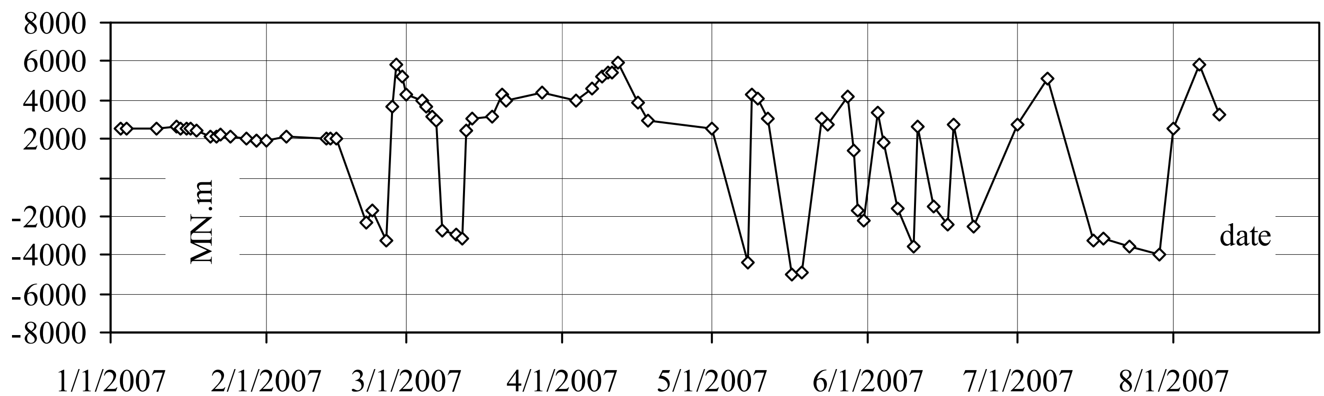

Fig. 5 M_{SW} distribution for Jan. 2007-Aug. 2007

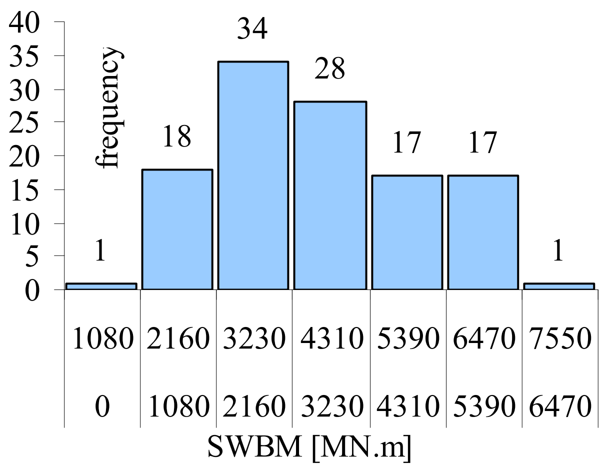

Fig. 6 Histogram of the hogging still water bending moments (116 records)

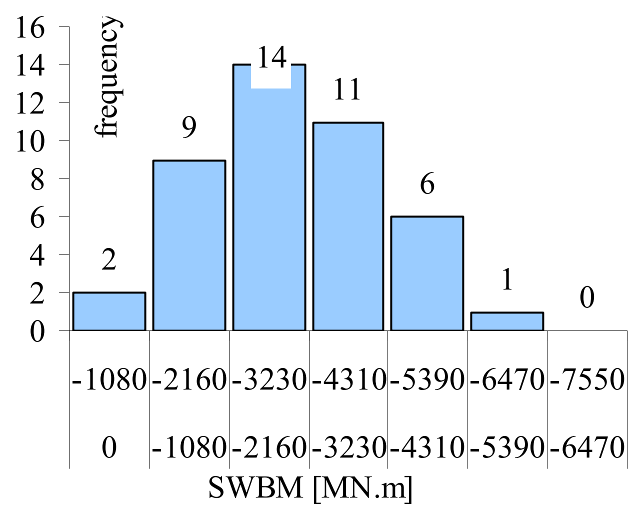

Fig. 7 Histogram of the sagging still water bending moments (43 records)

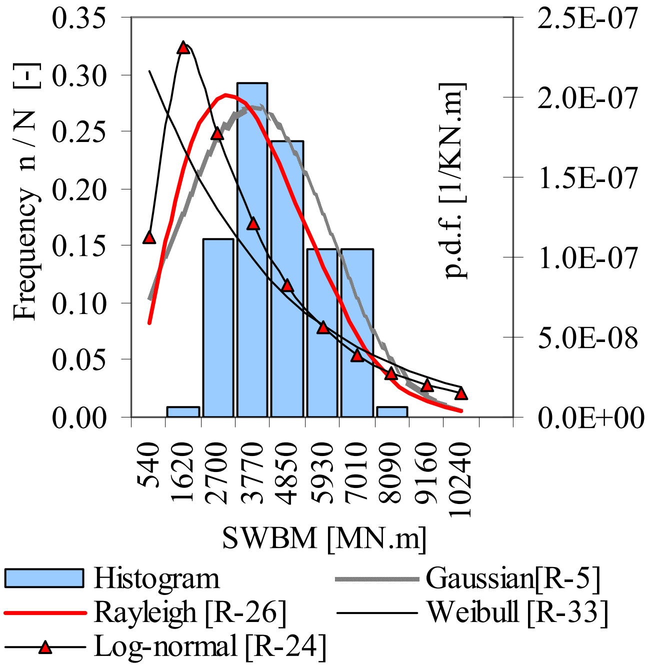

Fig. 8 Histogram of hogging M_{SW} and several p.d.f. that fit the statistical data

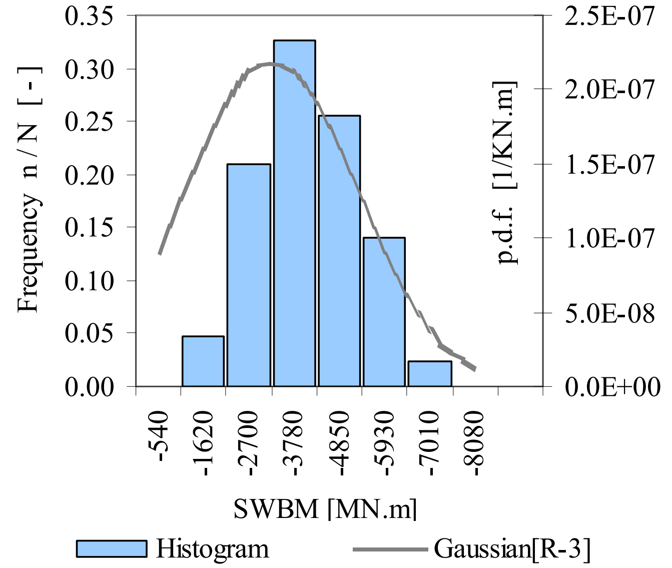

Fig. 9 Histogram of sagging M_{SW} and Gaussian p.d.f. that fit the statistical data

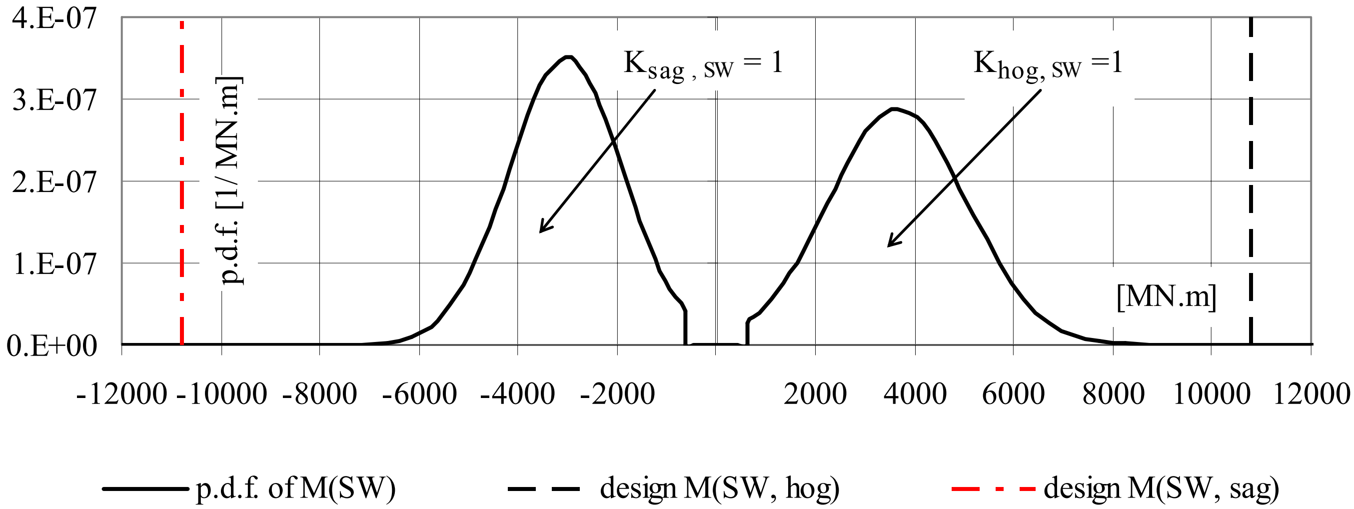

Fig. 10 Truncated Gaussian distribution of M_{SW} when hogging and sagging are treated separately

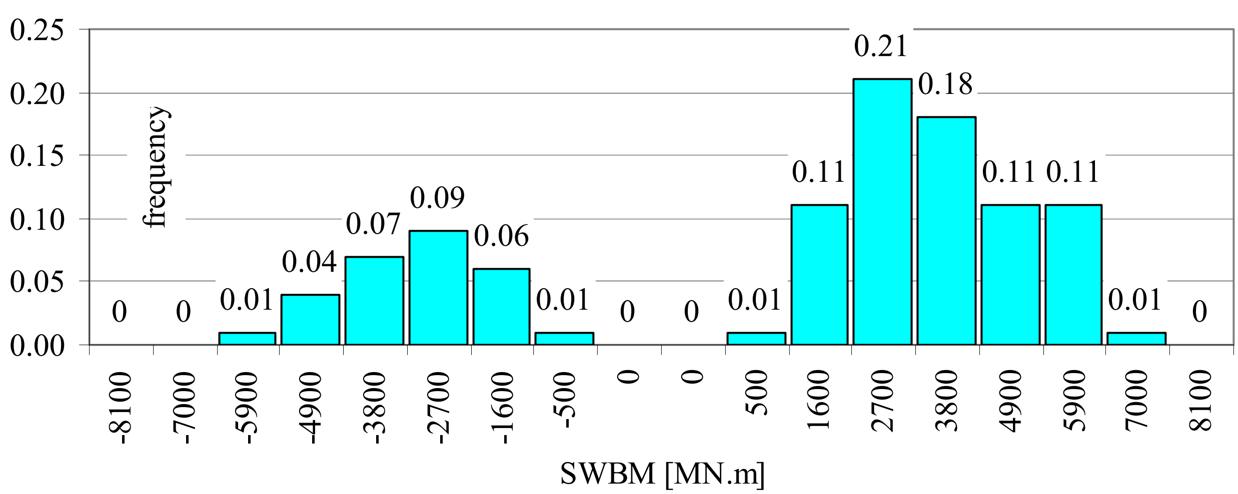

Fig. 11 Histogram of sagging and hogging M_{SW} considered as two sides of one phenomenon

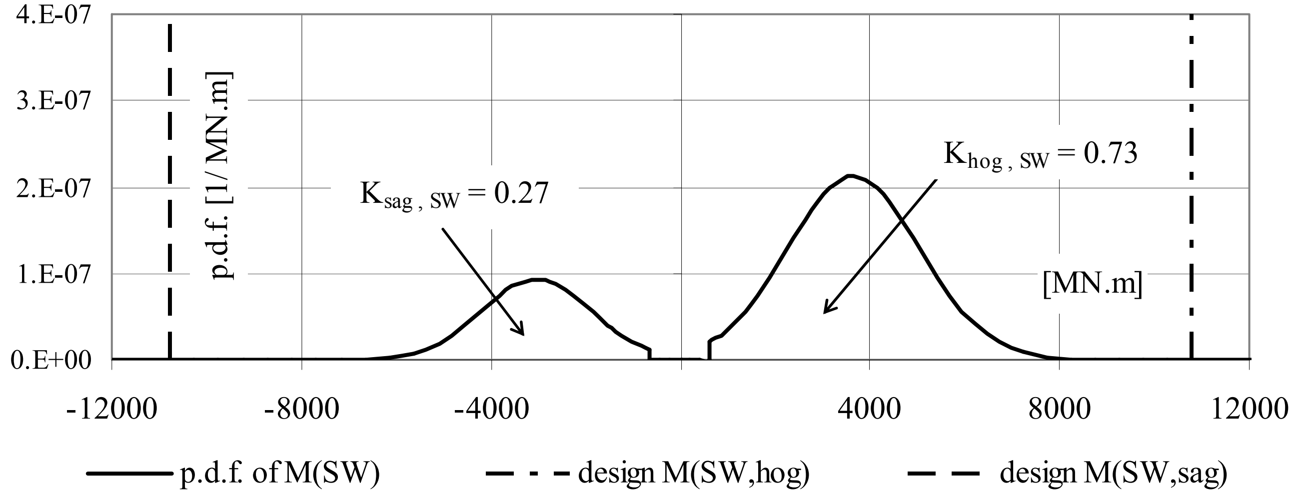

Fig. 12 Truncated Gaussian distribution of M_{SW} when hogging and sagging are treated as two sides of one phenomenon

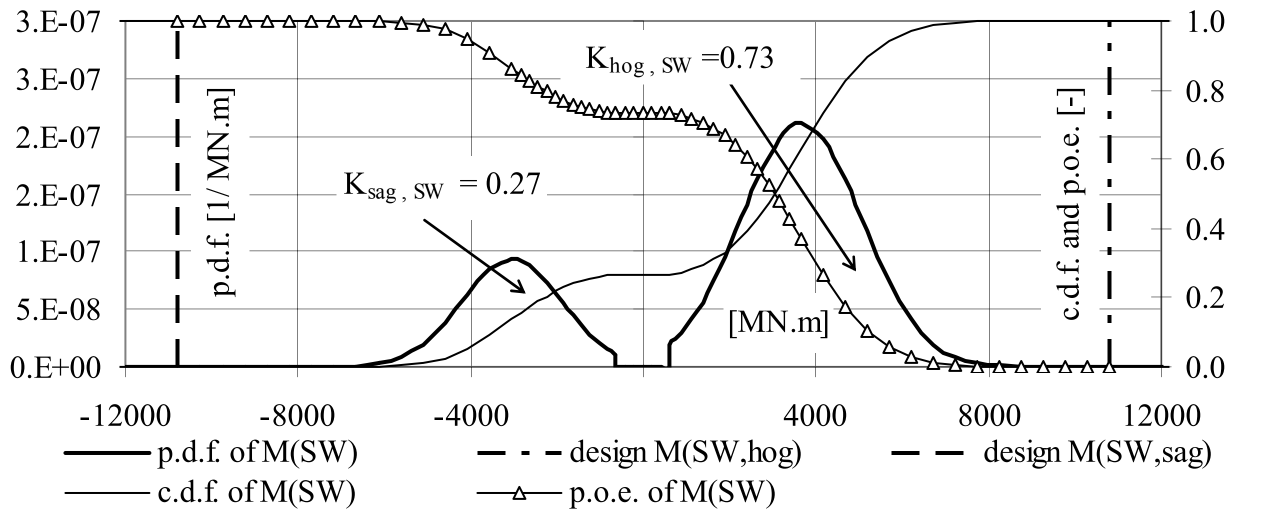

Fig. 13 Truncated Gaussian distribution for hogging and sagging M_{SW} derived by individual amplitude statistics while treating the data for hogging and sagging as two sides of one phenomenon (design $M_{SW} = \pm 10780390$ KN.m)

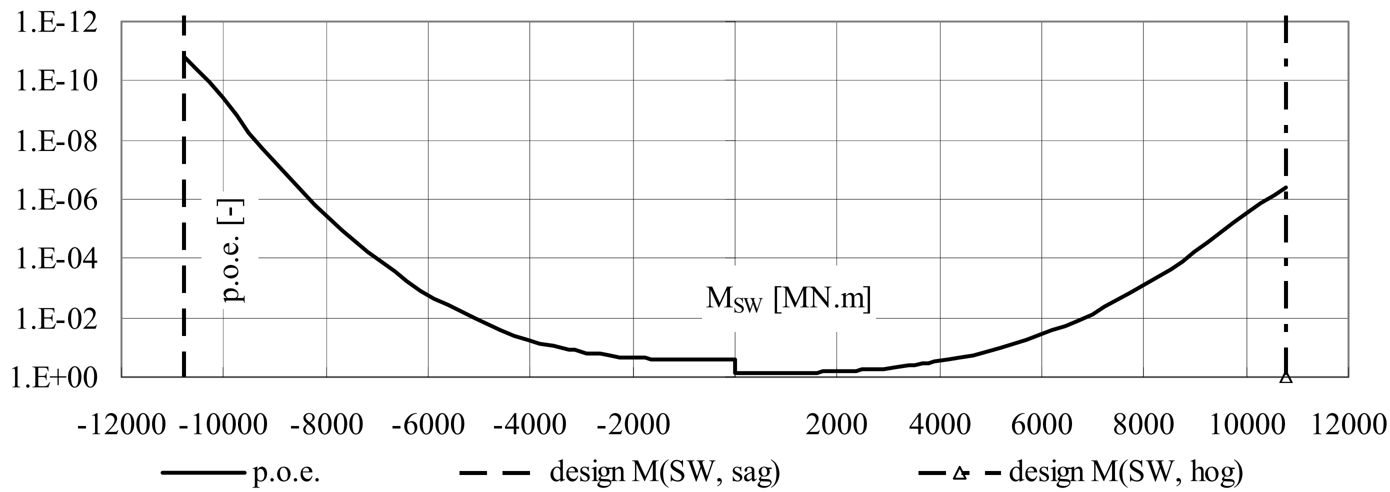

Fig. 14 P.o.e. of M_{SW} in logarithmic scale (individual amplitude statistics are used)

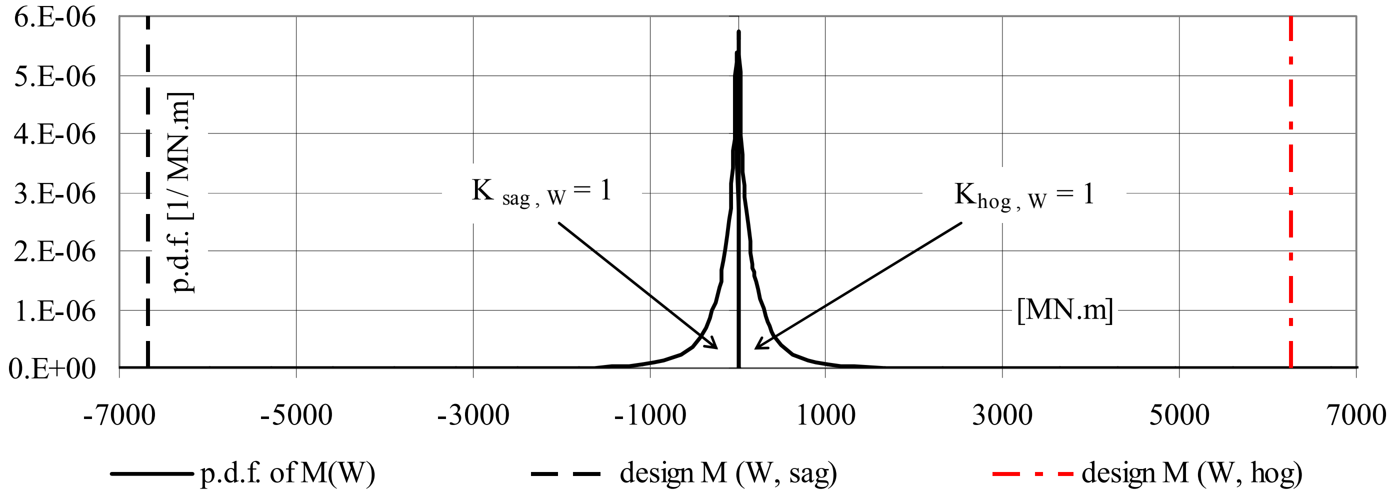

Fig. 15 Weibull p.d.f. of M_W when hogging and sagging are treated separately (individual amplitude statistics are applied)

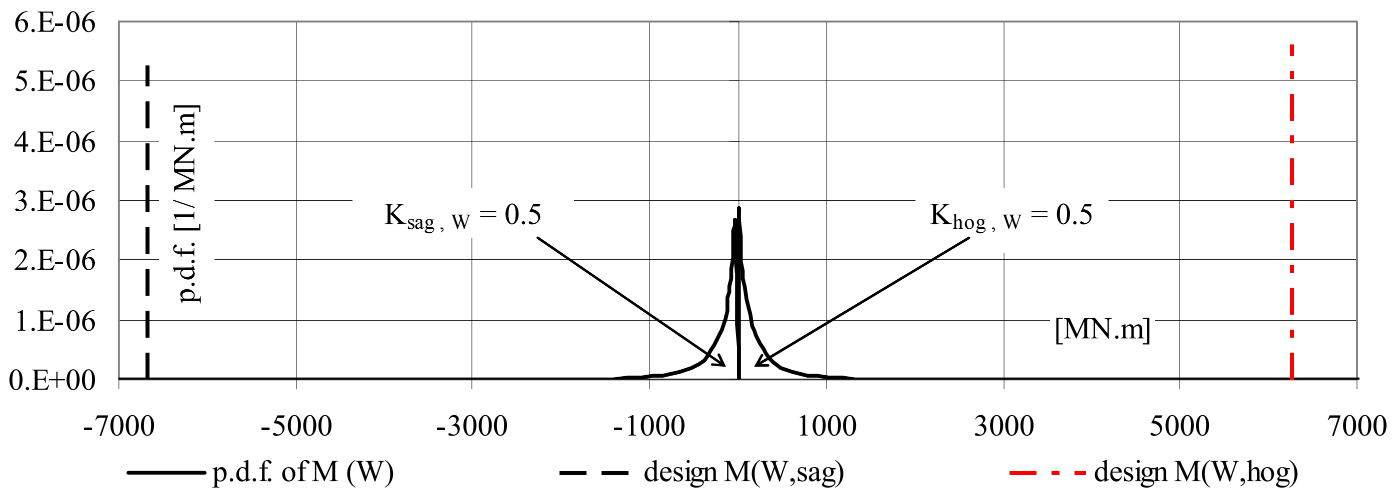

Fig. 16 Weibull p.d.f. of M_W when hogging and sagging are treated as two sides of one phenomenon (individual amplitude statistics are applied)

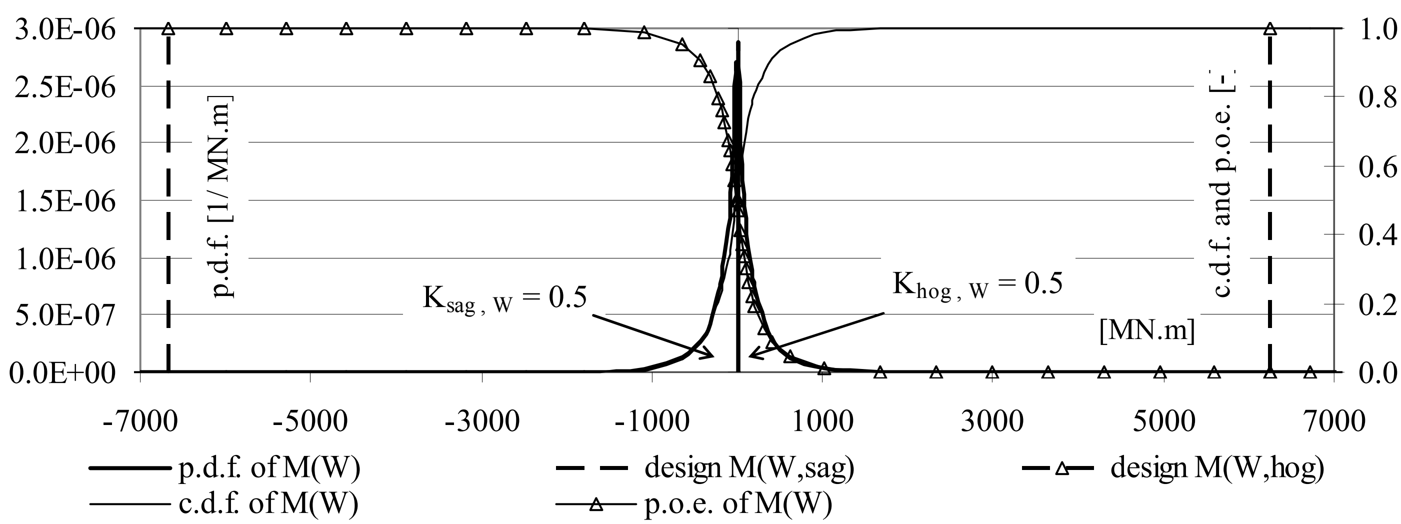

Fig. 17 Distribution functions of M_W when hogging and sagging are treated as two sides of one phenomenon (individual amplitude statistics are applied). The design M_W for hogging is 6258260 [KN.m] and -6678920 [KN.m] for sagging

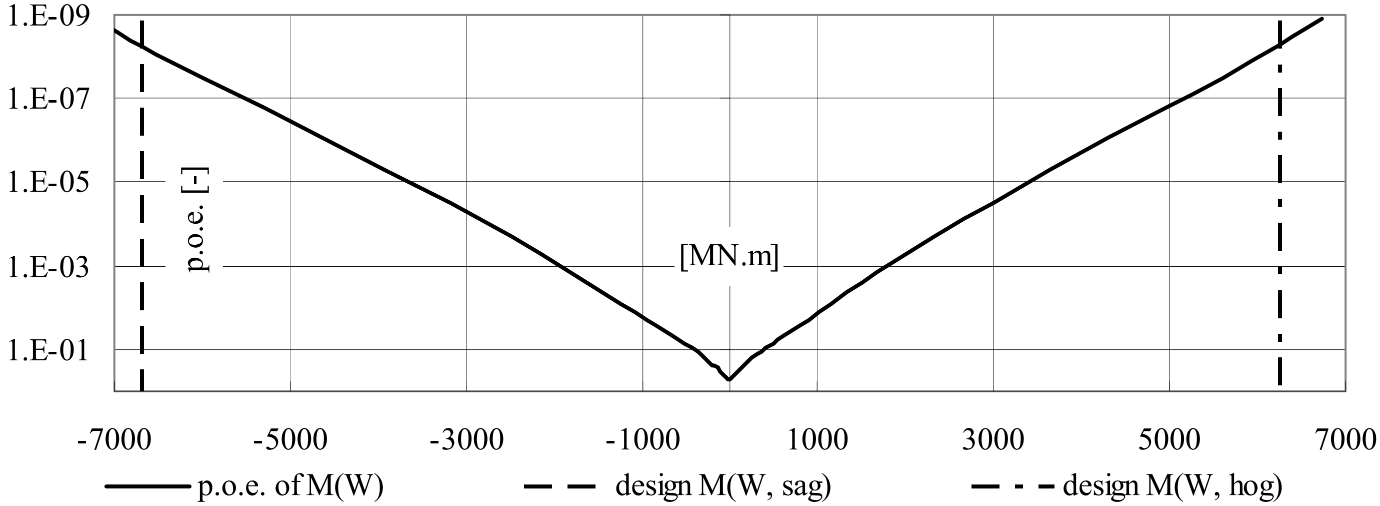

Fig. 18 P.o.e. of M_W when hogging and sagging are treated as two sides of one phenomenon in log-scale (individual amplitude statistics are applied)

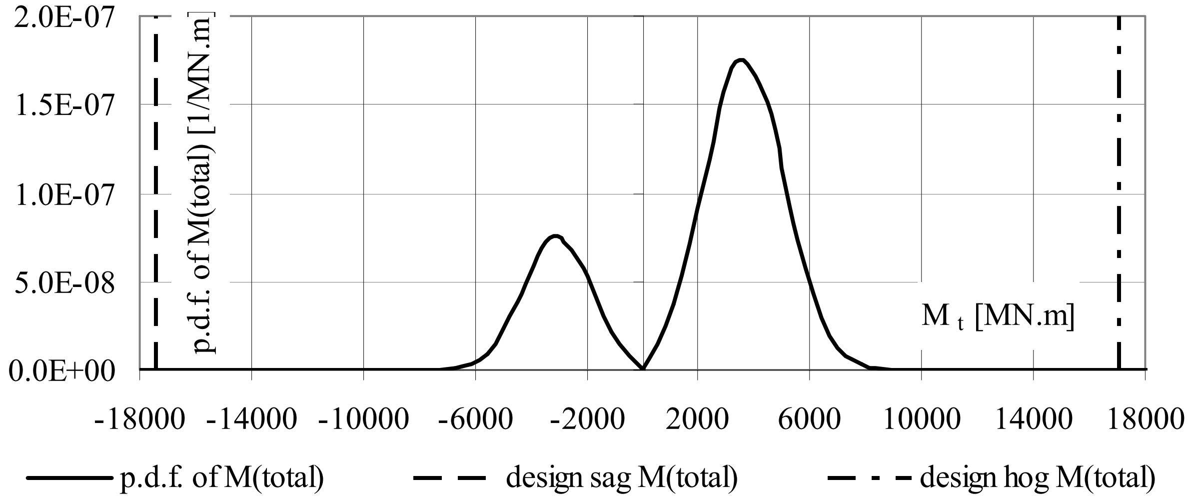

Fig. 19 P.d.f. of M_t obtained by differentiating Eq. (16); individual amplitude statistics are used; design M(total, sag) = -17459310 [KN.m]; design M(total, hog) = 17038660 [KN.m])

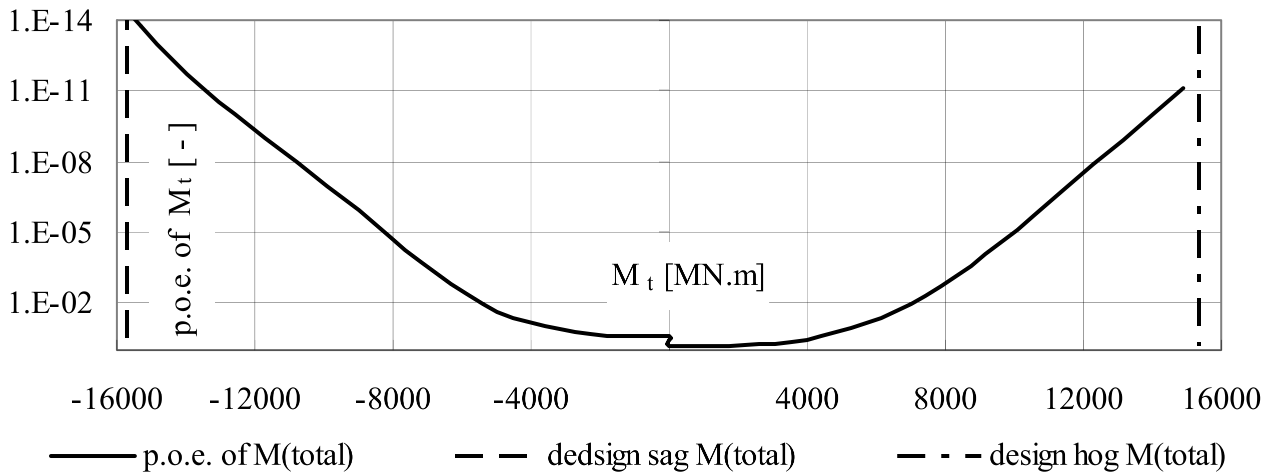

Fig. 20 P.o.e. of M_t in log scale and the design values of M_t (individual amplitude statistics are applied)

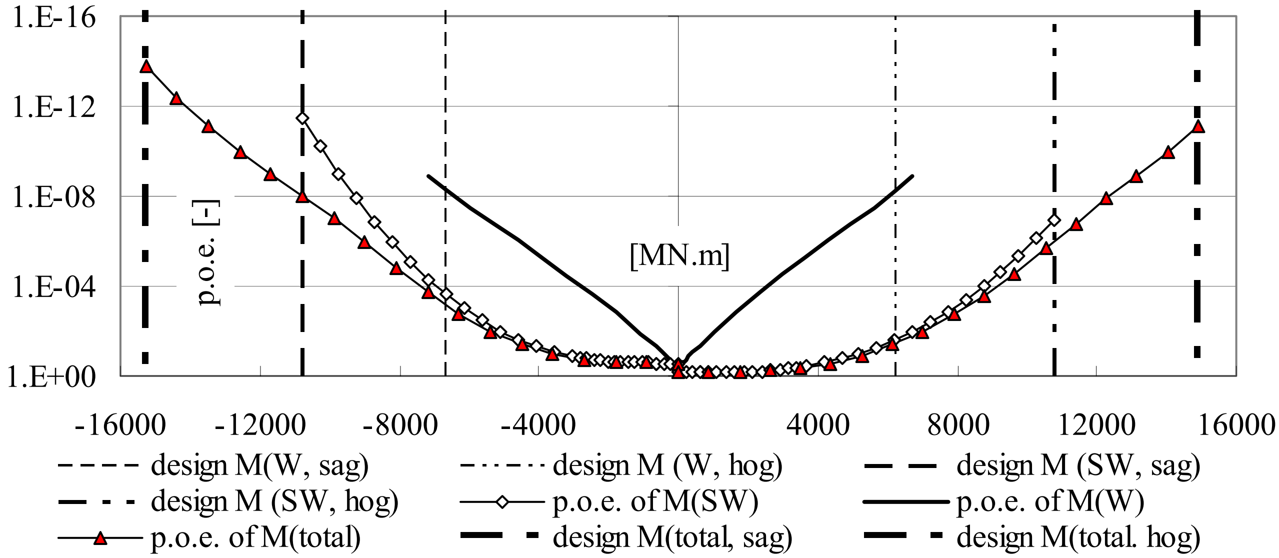

Fig. 21 P.o.e. of M_{SW}, M_W and M_t in log scale and their design values (individual amplitude statistics are applied). The coefficient ϕ in Eq. (20) is 0.875

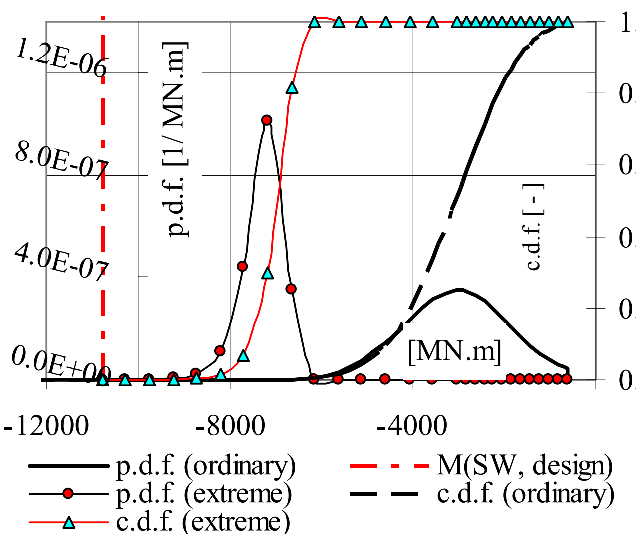

Fig. 22 P.d.f. and c.d.f. of sag M_{SW} obtained by extreme value statistics and individual amplitude statistics

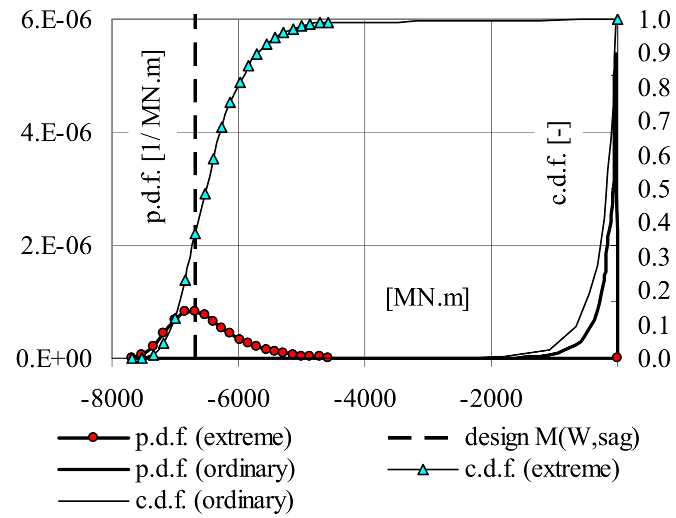

Fig. 24 P.d.f. and c.d.f. of sag M_W obtained by extreme value statistics and individual amplitude statistics

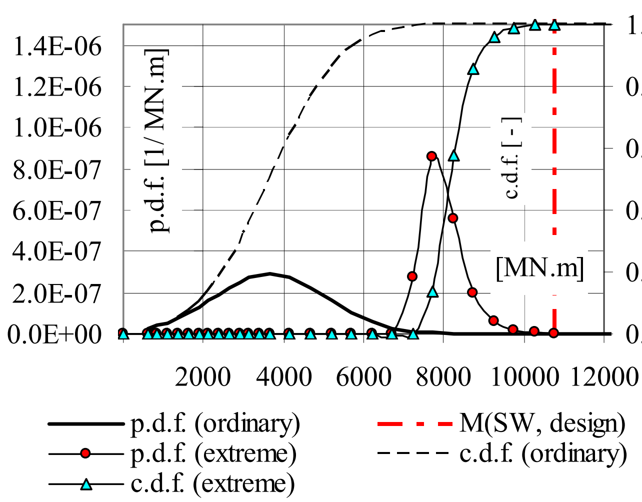

Fig. 23 P.d.f. and c.d.f. of hog M_{SW} obtained by extreme value statistics and individual amplitude statistics

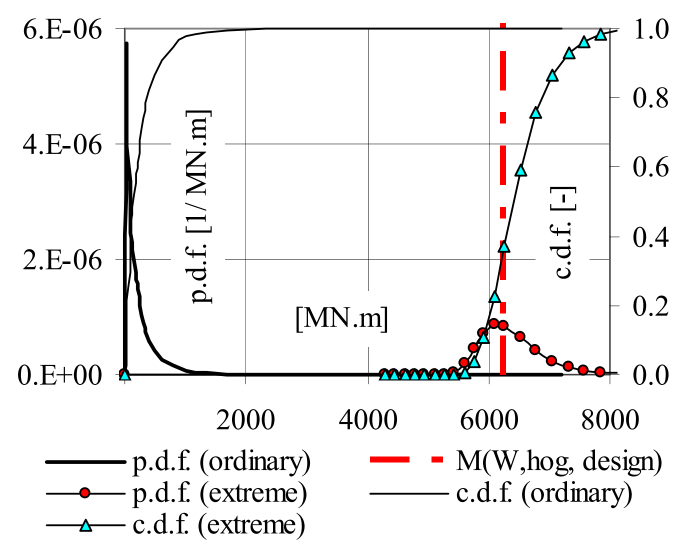

Fig. 25 P.d.f. and c.d.f. of hog M_W obtained by extreme value statistics and individual amplitude statistics

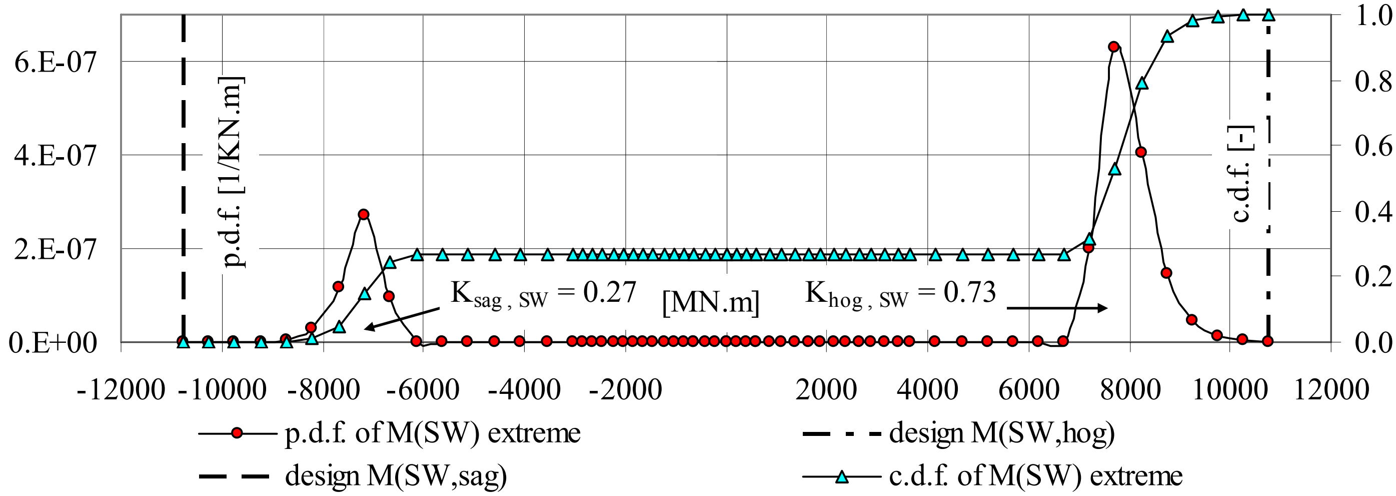

Fig. 26 Probabilistic distributions of M_{SW} when sagging and hogging are treated as two sides of one phenomenon (extreme value statistics are applied)

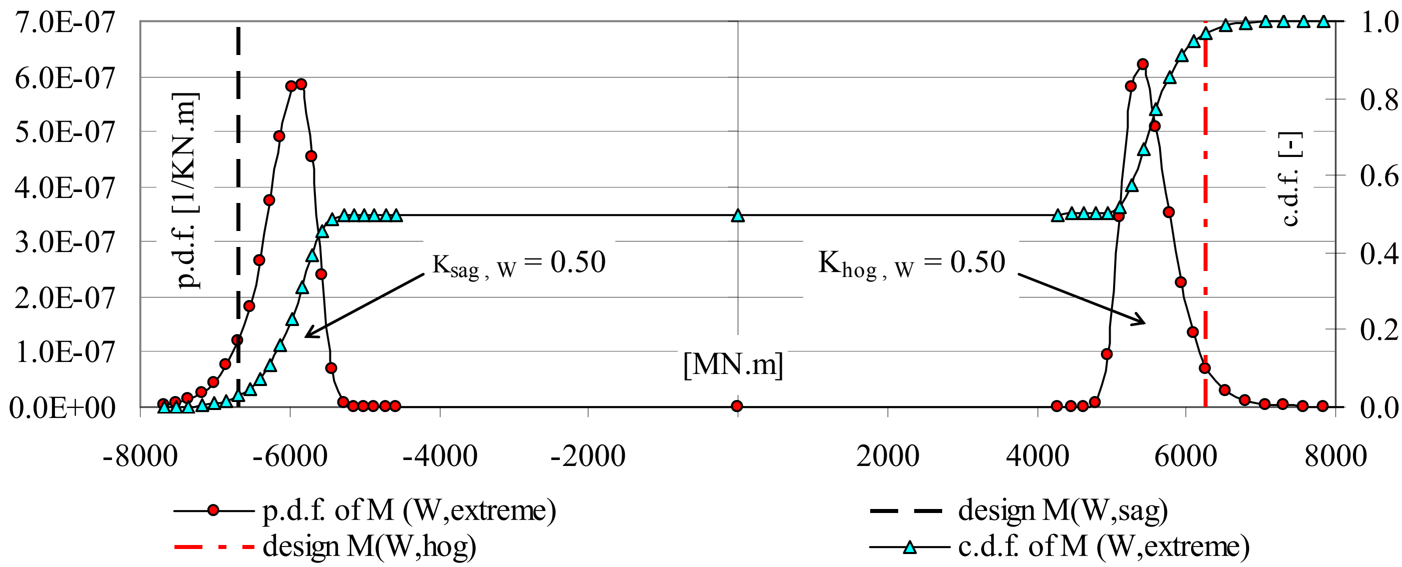

Fig. 27 Probabilistic distributions of the extreme value of M_W when sagging and hogging are treated as two sides of one phenomenon

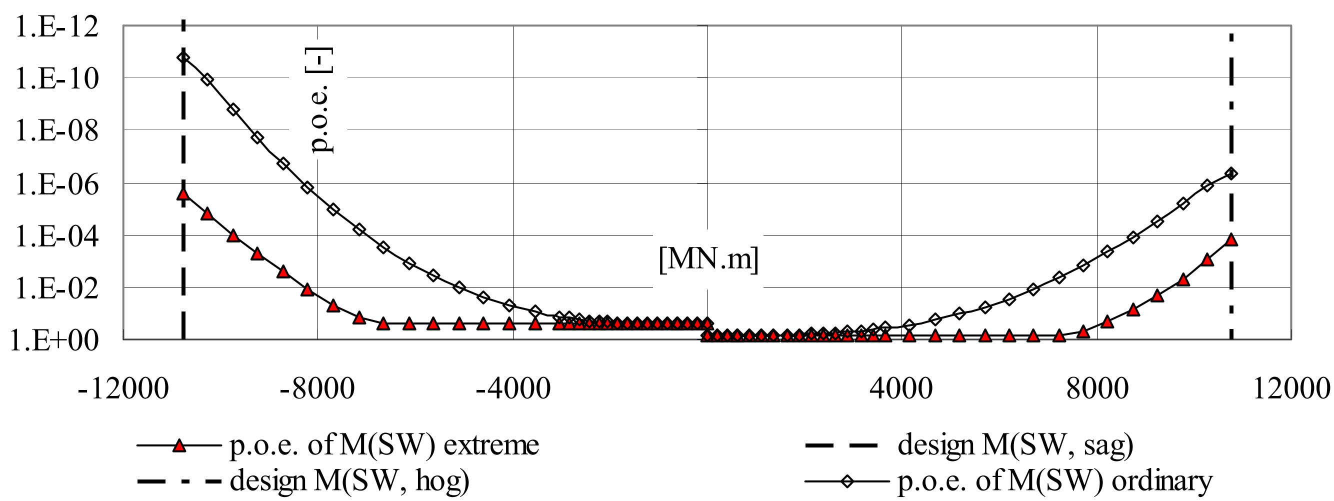

Fig. 28 P.o.e. in log-scale of M_{SW} when hogging and sagging are treated as two sides of one phenomenon (extreme and individual amplitude statistics are applied)

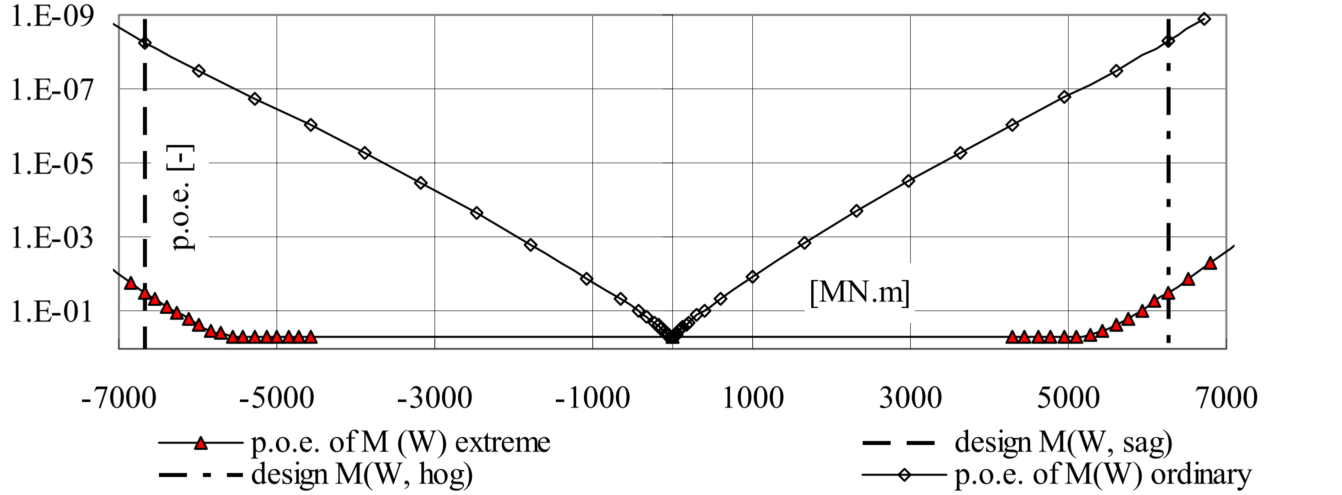

Fig. 29 P.o.e. in log-scale of M_W when hogging and sagging are treated as two sides of one phenomenon (extreme and individual amplitude statistics are applied)

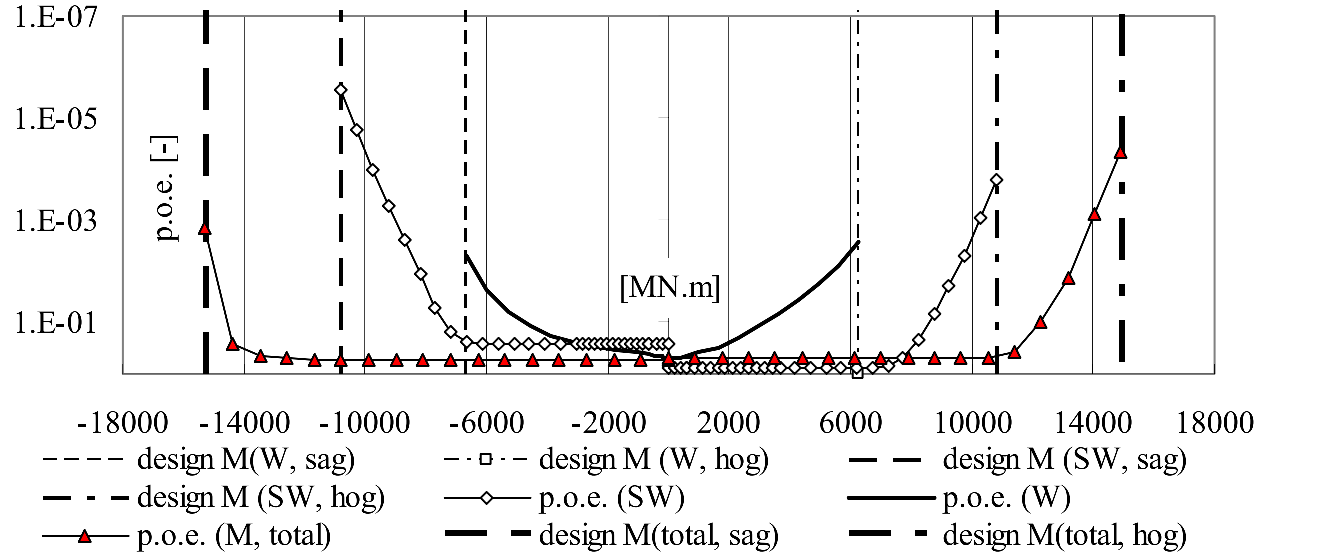

Fig. 30 P.o.e. of M_{SW}, M_W and M_t in logarithmic scale when sagging and hogging are treated as two sides of one phenomenon (extreme value statistics are applied).

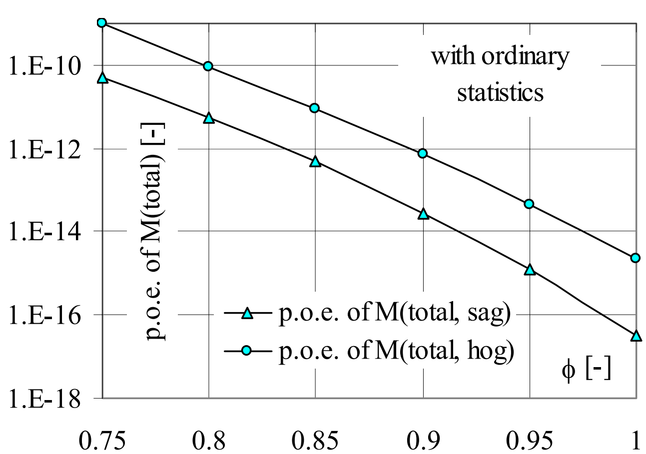

Fig. 31 P.o.e. of any given limit value of M(total, sag) presented as a portion of (M_{SW} + M_W) by the coefficient ϕ (see Eq. (20))

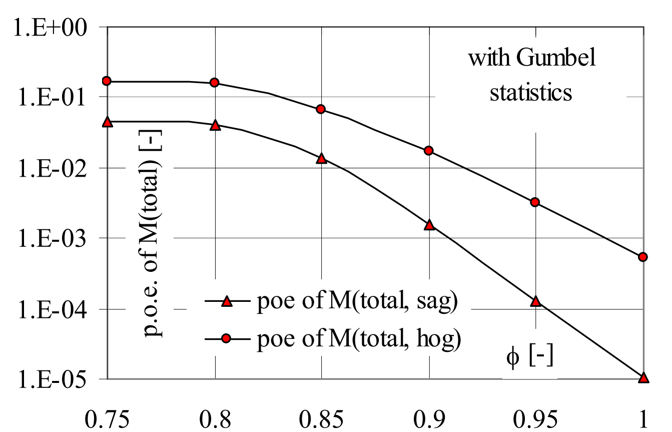

Fig. 32 P.o.e. of any given limit value of M(total, hog) presented as a portion of (M_{SW} + M_W) by the coefficient ϕ (see Eq. (20))