Calculation of Reliability-Based Safety Factors for Establishing Defect Acceptance Criteria for Deepwater Riser Welds

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Abstract

This paper presents a reliability-based methodology to determine appropriate safety factors for establishing defect acceptance criteria for riser weld design. These criteria need to be established for weld inspections to ensure riser safety.

The allowable defect size is determined using fracture mechanics, e.g., the methods described in British Standard BS 7910 [1]. Similar to SN fatigue calculations, safety factors are applied to the wave-induced and VIV-induced stress range histograms. However, in contrast to the SN procedure, additional safety is introduced through conservative selection of the many other parameters used in the fracture mechanics calculations. Due to the absence of an industry standard calibrated approach, the failure probability associated with weld defects is expected to vary significantly from riser to riser.

The scenarios of either unsafe or excessively conservative design are both of significant concern. In the former case there is an increased risk of riser failure. In the latter case, the unduly stringent criteria may result in unnecessary weld cut-outs or may become impractical relative to available inspection methods, thereby compromising riser feasibility.

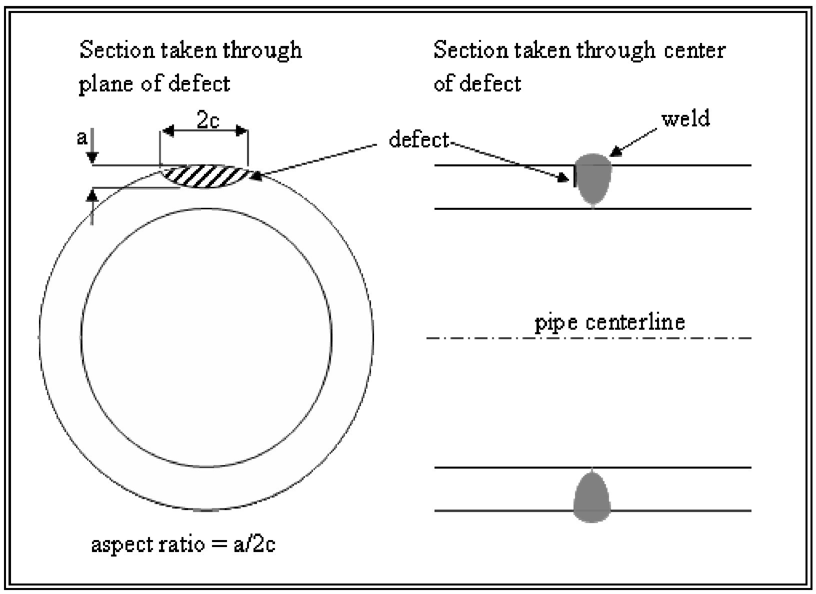
Through practical examples, this paper demonstrates that the proposed reliability method can be used to determine appropriate safety factors that ensure the safe and economic design of deepwater riser welds.

Introduction

This paper discusses the determination of appropriate safety factors to be used in the Engineering Criticality Assessment (ECA) for the welding of riser joints. The ECA evaluation gives allowable crack sizes to be applied in weld inspections. If during weld inspections cracks with sizes larger than allowed were found, the weld must be rejected.

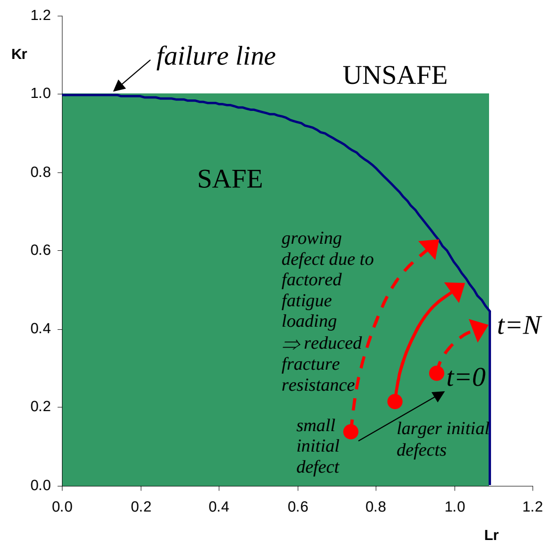
The defects of concern are located at the pipe-to-pipe girth weld toe, and for the purposes of assessment are assumed to lie in a plane normal to pipe axis and to be elliptical in shape as shown in Figure 1.

Figure 1 – Schematic of Defect



Engineering Criticality Assessment (ECA) is the terminology used by BS7910 for the procedure described therein for establishing the fitness-for-purpose of metallic structures (and in particular welds) with known defects. For known defect geometry, material properties and load conditions, the fracture resistance of a weld can be assessed using the Failure Assessment Diagram (FAD) approach, shown schematically in Figure 2. The fracture load normalized by the fracture resistance (Kr) and the primary stress normalized by the material flow stress (Lr) are calculated to provide a design point. The design point is plotted on the FAD to determine whether the defect is safe or unsafe. For structures that are subjected to fatigue loading, the defect will grow with time until it reaches an unsafe size. The purpose of ECA for riser welds is to establish initial defect sizes that are sufficiently small that they will remain safe throughout the riser service life.

Figure 2 – Typical Failure Assessment Diagram



Reliability analysis is used to calculate the *probability* of an allowable defect becoming through-thickness due to a combination of sub-critical growth due to fatigue loading and the fracture due to an extreme event. The random variables considered in the reliability analysis include the following: initial crack size, crack growth parameters, wave and VIV induced stress uncertainty factors, pipe wall thickness, annual extreme stress from environmental load and FAD diagram conservatism. The safety factor pair (one for wave induced stress range and the other for VIV induced stress range) is applied by increasing the stress range cycles during the ECA crack propagation calculations.

The qualitative behavior of failure probability with different safety factors is illustrated in Figure 3. For small safety factors, relatively large initial cracks are allowed and the failure probability is expected to increase sharply as safety factors become increasingly smaller. For high safety factors, the riser failure probability reaches a constant value. This is due to the fact that for high safety factors, only very small initial cracks are allowed and hence the failure probability can be approximated by the plastic ultimate strength failure of an un-cracked cross section. The objective of this study is to demonstrate a reliability methodology which can produce results as shown in Figure 3 via an example application, and hence can help decision making on selections of safety factors as depicted in the shaded zone in Figure 3.

In addition to developing the methodology, a set of study cases was evaluated to test the methodology and quantify the likely safety factor requirements. A typical deepwater SCR was used as the study base, with the stress ranges scaled such that the S-N fatigue life was exactly 30 years for all study cases. Four cases were examined:

1. Touch-Down Point (TDP), crack at outer surface with 1/4 aspect ratio.
2. TDP, crack at outer surface with 1/20 aspect ratio.
3. Top-Weld (TW) region, crack at outer surface with 1/4 aspect ratio.
4. TW region, crack at outer surface with 1/20 aspect ratio.

These four cases were chosen to cover the two most stressed regions (TDP and TW), as well as two defect aspect ratios. The two most stressed regions are illustrated as zones 1 and 4 as shown in Figure 4.

Figure 3 – Illustration of Failure Probabilities with Varying Safety Factors

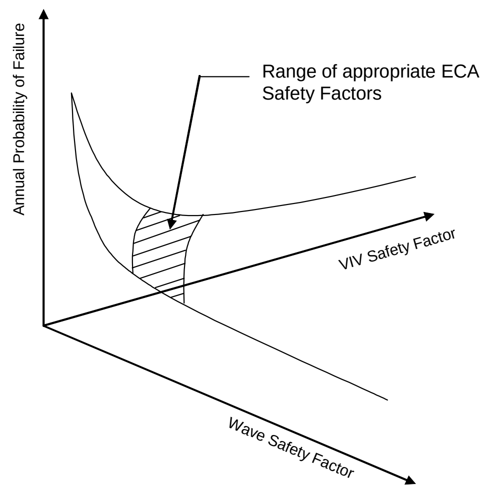
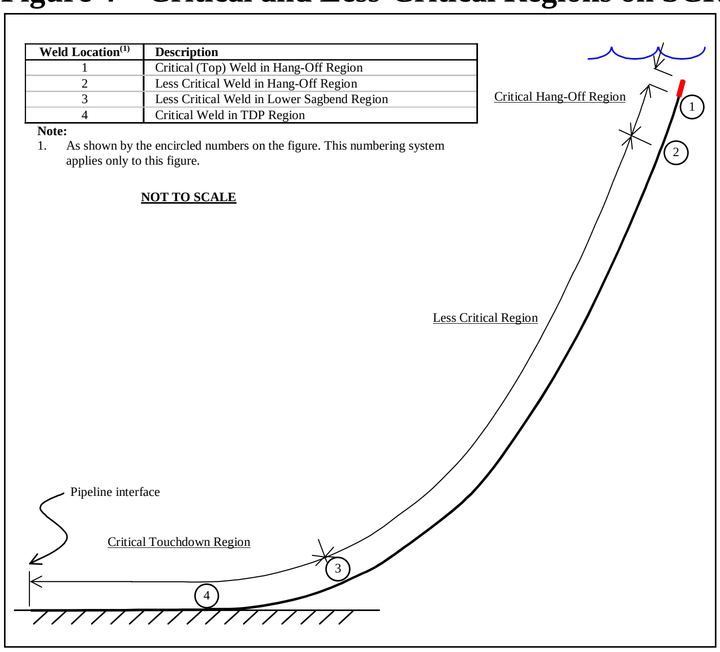


Figure 4 – Critical and Less-Critical Regions on SCR

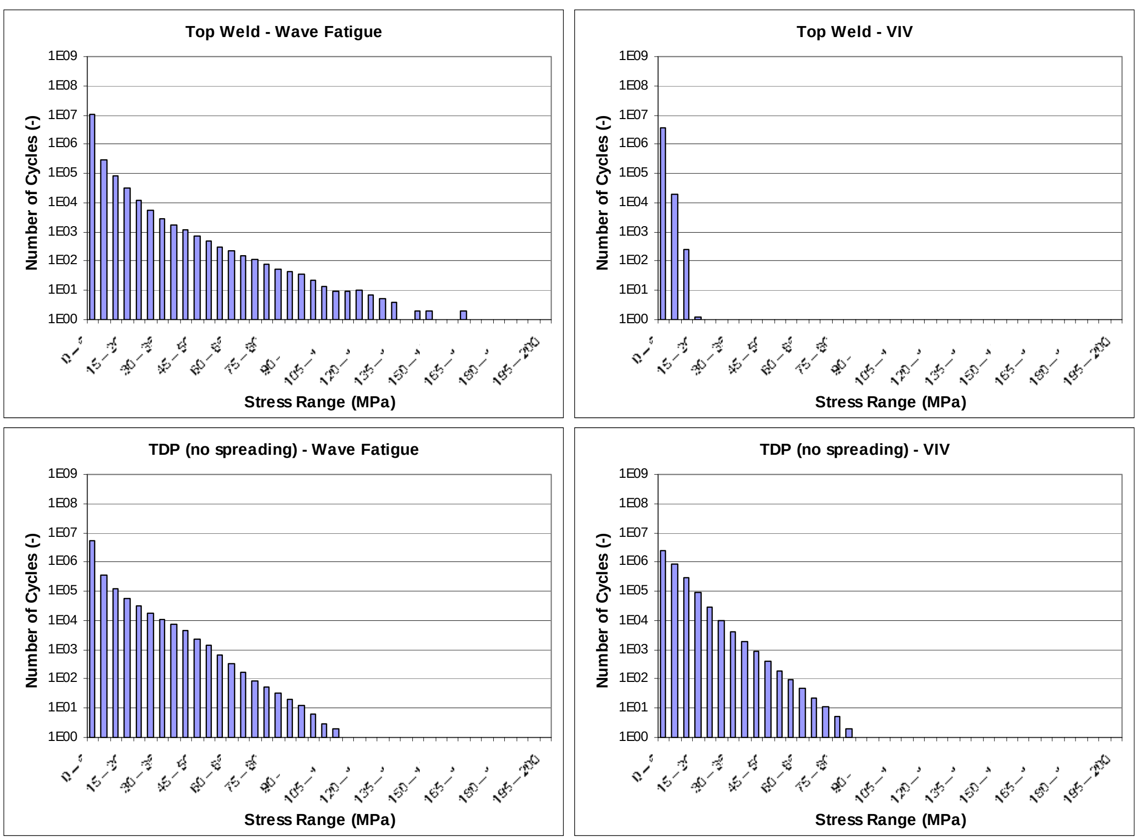


Study Cases and Approach

The study cases were derived from an actual deepwater Gulf of Mexico SCR, with some modifications made in order to keep the study reasonably generic. Some of the notable features and differences between the two locations are described below. These are considered to be relatively generic for SCRs supported from semi-submersible platforms.

- The wave loading fatigue histogram for the top weld location is concave, relative to that for the touchdown region, with greater high stress-range cycles and fewer mid stress-range cycles, see Figure 5.
- VIV fatigue in the touchdown region may be similar in to wave fatigue, whereas VIV fatigue at the top weld is negligible.
- The predicted extreme longitudinal tensile stress is greater for the lower region than for the top weld.
- The position of the top weld is known and it is apparent that the extreme stress in this region will coincide with the location of maximum fatigue loading. In contrast, the touchdown point of the riser moves as the platform moves. It is likely that an extreme event maximum stress will occur at a different location to that where the majority of fatigue-damage accrues.

Figure 5 – Unfactored Annual Stress Range Histograms for Study Cases



The four original stress range histograms are presented in Figure 5. For the purposes of this study, the stress range histograms were adjusted by scaling the stress range while maintaining the number of cycles until the S-N fatigue life criteria was exactly satisfied. Therefore, the study case histograms correspond to the case where the weld cap S-N fatigue life is designed to the limit. The BS 7608: 1993 D class S-N curve [2] was used. For simplicity, no correction was made for exposure to seawater, and the slope change for high cycle fatigue was not applied. For both locations, only the OD surface defects were assessed, as this has been found to be the most critical location with respect to defect acceptance criteria.

The overall approach carried out in this study is summarized as follows:

- Step 1 – Normalize stress ranges for all four study cases such that they produce a S-N based fatigue life of 30 years.
- Step 2 – Calculate allowable initial crack sizes using ECA approach for different combination of safety factors for wave and VIV, e.g., (1,1), (5,5), (5,10), etc.
- Step 3 – Based on Step 2, calculate initial crack size probability distributions after weld inspection incorporating sizing errors, for each safety factor pair.
- Step 4 – Prepare input data for other random variables, which includes crack growth parameters, misalignment, wall thickness, etc.
- Step 5 – Perform response-surface reliability calculations.
- Step 6 – Iterate Step 5 until reliability solutions converge.
- Step 7 – Post-process annual failure probabilities for each case.

The required ECA and structural reliability approach is explained in more details as follows.

ECA Methodology

The ultimate capacity of a structure may be significantly reduced by the presence of a macroscopic crack, depending upon the toughness of the material, the geometry and location of the crack, and the nature of the loading. Fracture mechanics is the branch of engineering concerned with the analysis of cracked structures or components.

Engineering Criticality Assessment (ECA) is the terminology used by BS7910 for the procedure described therein for establishing the fitness-for-purpose of metallic structures (and in particular welds) with known defects. Fracture resistance and fatigue are discussed in turn below, followed by a brief discussion on the derivation of the defect acceptance criteria, which is the ultimate objective of the ECA performed during SCR detailed engineering.

<u>Fracture Resistance.</u> Fracture resistance refers to the ability of a material to resist the unstable propagation of an existing crack under an applied load condition. An important parameter for assessing fracture resistance is the linear stress intensity, K, which is a measure of the magnitude of the stress field in the vicinity of the crack tip. Theoretically, if the stress intensity at the crack tip exceeds the material toughness, K_{mat}, then unstable crack growth will occur. The ratio $K_r = K/K_{mat}$ is defined as the fracture ratio. The limit state is given by $K_r = 1$.

Plastic failure, rather than fracture, may be the dominant failure mode. Therefore, it must be confirmed that the applied load contributing to plastic failure, σ_{ref}, does not exceed the plastic failure resistance, σ_{flow}, for the flawed structure. Here $\sigma_{flow} = (\sigma_u + \sigma_y)/2$, where σ_u is the ultimate tensile strength and σ_y is the yield strength of the material. The ratio $L_r = \sigma_{ref} / \sigma_y$ measures the propensity for plastic failure. Note that in this case $L_r = L_{r\_max}$ is the ultimate limit, where $L_{r\_max} = \sigma_{flow}/\sigma_y$.

There is interaction between the two failure mechanisms, which is addressed in BS 7910 by the failure assessment diagram (FAD), as shown in Figure 2. The separate criteria for fracture toughness and plastic failure are replaced by an interaction criterion, whereby the inequality $K_r < f(L_r)$ must be satisfied. For the general (or Level 2A) FAD, the failure line is defined as follows:

$$\begin{aligned} K_r &= \left(1 - 0.14 \cdot L_r^{\,2}\right)\left\{0.3 + 0.7\exp\left(-0.65 \cdot L_r^{\,6}\right)\right\} \text{ for } L_r \le L_{r\_\max} \\ &= 0 \quad \text{for } L_r > L_{r\_\max} \end{aligned} \tag{1}$$

Sub-Critical Crack Growth. Under cyclic load conditions crack growth rate is related to the associated stress intensity range at the crack tip, by the empirical relationship:

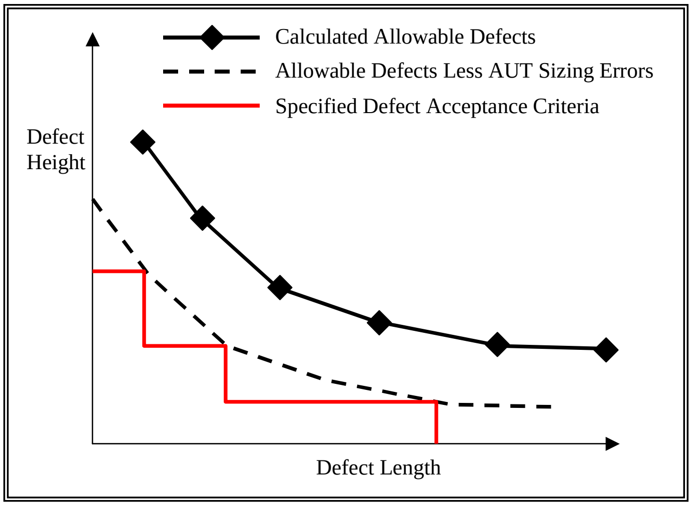
$$da / dN = A \cdot \left(\Delta K\right)^m \tag{2}$$

A and m are constants that depend on the material and the environment, and (ΔK) is the stress intensity range at the crack tip.

Defect Acceptance Criteria. As discussed in the introduction, the objective of prescribing defect acceptance criteria is to ensure that defects that pass as acceptable will not grow through-thickness during the service life of the riser. During its service life, the riser will be exposed to continuous cyclic loading due to vessel motions, waves, and current. In addition, the riser will be subjected to less frequent "extreme" load conditions, e.g., due to hurricane events.

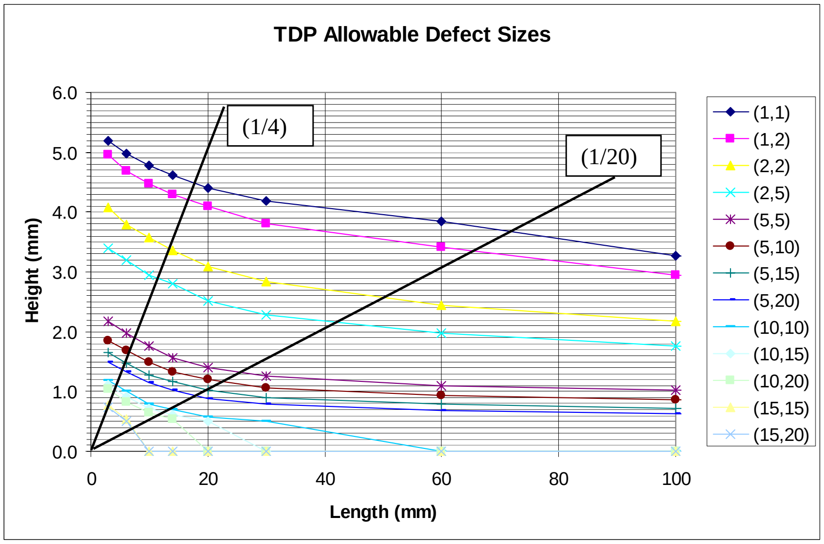
Defect acceptance criteria are developed using both fatigue and extreme loading. For a given allowable initial defect height, the maximum allowable length is estimated based on experience. The defect is then grown in accordance with the crack growth methodology described above using the factored annual fatigue loading history. The fracture resistance is checked for the ever-growing defect after each year's worth of factored fatigue loading, using the governing extreme load condition. This process continues until either unstable fracture is predicted or the service life for the riser is satisfied. The initial defect length is adjusted iteratively until the service life is just satisfied. This whole process is then repeated for several initial defect heights, to obtain a curve of allowable initial defect height versus allowable initial defect length. For specifying the criteria to be applied, sizing errors are then subtracted from the length and height of the allowable defect size curve. The defect acceptance criteria are then established on the basis of this latter curve. Typically the final acceptance criteria take the form of a step function, which can easily be tabulated, to help avoid misinterpretation in the field. The procedure is shown schematically in Figure 6.

Figure 6 – Specifying Defect Acceptance Criteria



The calculated allowable defect sizes for the study case (TDP region) are shown in Figure 7, for different safety factor pairs. The data were interpolated to obtain defect sizes with the study aspect ratios, as shown.

Figure 7 – Allowable Crack Sizes for the TDP Region



Reliability Analysis

The structural reliability calculation determines the probability of a structural failure event. A structural failure event is defined by a mathematical function (termed the limit-state function, or *g*-function). The limit-state function in the current study is defined by the position in the FAD diagram. When the ECA calculation determines a position inside the FAD diagram, the limit-state function returns a positive value signifying a successful event. A failure event is represented by a limit-state function with negative values, with a position outside of the FAD diagram. With reference to Figure 8, the limit-state function, g, is defined by the following equation:

$$g = d_2 - d_1 \tag{3}$$

where

- d_1 is the radial distance from the origin to the ECA evaluation point on the K_r, S_r (or L_r) plane. d_1 is a function of random variables x_1 to x_9 as listed in Table 1,
- d_2 is the radial distance, with the same angle from axis S_r (or L_r) as line d_1, to the surface of FAD curve with the addition of conservatism. d_2 is a function of random variable x_{10} as listed in Table 1.

It can be readily verified that $g > 0$ when $d_2 > d_1$ and represents successful events, and vice versa.

Figure 8 – Definition of Limit-State Function for ECA Reliability

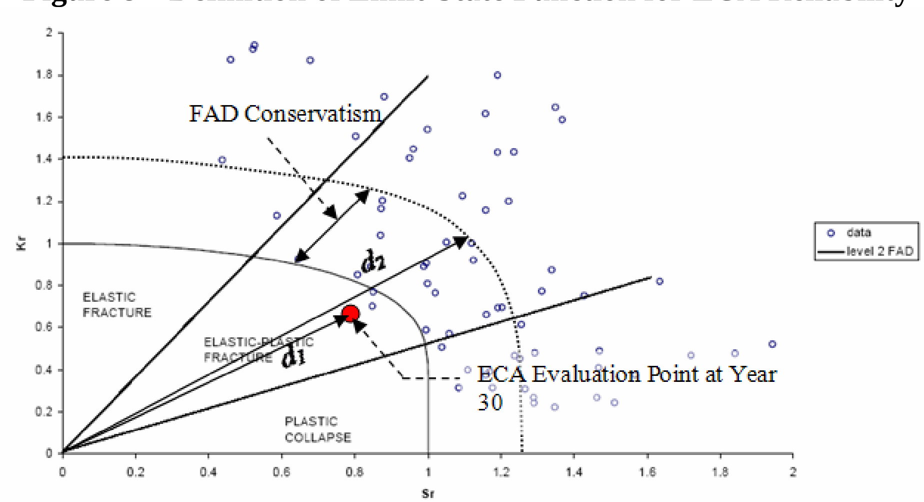


Table 1 shows the random variables used in reliability calculations, which applies to Study Cases 1 and 2. Random variables for Study Cases 3 and 4 differ only in the definition of variable x9 (annual extreme stress), to be explained later. The sources of these random variable definitions are explained as follows.

Initial Crack Size. Figure 9 shows the cumulative density function of initial crack size (height/depth) for Study Case 1, with Safety Factor pair (5,5). For this specific case, ECA calculation determines the allowable height to be roughly 1.4mm, after the deduction of sizing error allowances. Figure 9 conveys the message that after the weld inspection to filter out cracks with size greater than allowable, there is still approximately a 20% probability that a large crack (greater than allowable) might exist. This is derived based on reviews on AUT inspection trials of about 100 samples for a BP GOM deepwater riser project.

Table 1 – List of Random Variables for Study Cases 1 and 2, TDP

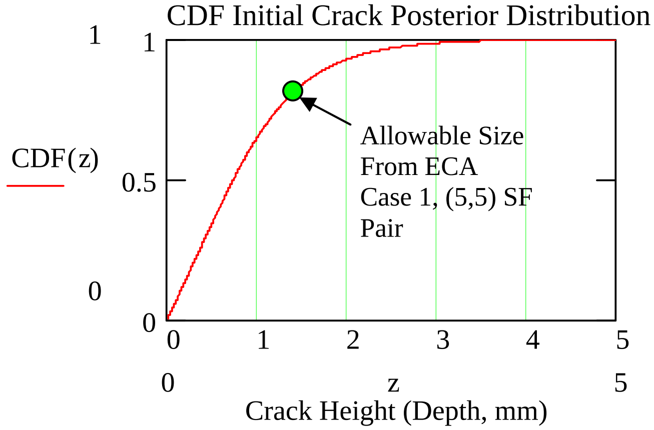
Random Variables	*Symbol*	*Basic Random Variable Numbering*	*Mean*	*COV*	*Sigma*	*Distribution Type*
Initial Crack (mm)	a_i	x1 (1,1)	2.526	79.7%	2.013	Composite
		(5,5)	0.884	79.8%	0.705	
		(5,10)	0.733	79.7%	0.584	
		(10,10)	0.379	79.7%	0.302	
		(10,15)	0.347	79.8%	0.277	
		(10,20)	0.303	79.9%	0.242	
Crack growth parameter stage1	Ln(A1)	x2	-39.878	-1.9%	0.738	Normal
Crack growth parameter stage 2	Ln(A2)	x3	-28.165	-1.4%	0.395	Normal
Misalignment (mm)	δ	x4	0.3330	70.8%	0.236	Triangular
Wave stress range modeling error	esw	x5	0.8670	25.3%	0.219	Log-normal
VIV stress range modeling error	esv	x6	0.6910	45.0%	0.311	Log-normal
Wall Thickness 1 (mm)	t1	x7	30.84	0.5%	0.1577	Normal
Wall Thickness 2 (mm)	t2	x8	30.84	0.5%	0.1577	Normal
Annual Extreme Stress (MPa)	σ	x9	233	25.0%	58.25	Log-normal
FAD Conservatism, Elastic Region	Ge	x10	1.62	51.2%	0.83	Weibull
FAD Conservatism, Elastic-Plastic Region	Ge-p	x11	0.47	104.3%	0.49	Weibull
FAD Conservatism, Plastic Region	Gp	x12	0.77	107.8%	0.83	Weibull

Crack Growth Parameter. The probability distributions for the two-stage crack propagations were taken directly from BS7910 [1].

Wave and VIV Stress Modeling Error. Following the study of Stahl and Banon [3], the stress modeling errors were taken with a bias factor (median value of the log-normal distribution) of 0.85 and a COV of 25% for wave-induced stress, and a bias factor of 0.63 and a COV of 45% for VIV-induced stress. Note that the median value differs slightly from the mean value for the log-normal distribution, as listed in Table 1.

Annual Extreme Stress. For the example Study Cases, the design extreme stress is 407MPa for TDP, and 341MPa for TW, which correspond to the 100-yr return period storm. This information was used, together with the 25% COV assumption (see above), to derive the mean value of the annual extreme stress. This derived log-normal distribution has a 1/100 probability of exceedence at 407MPa for TDP and 341MPa for TW, respectively.

Figure 9 – Cumulative Density Function of Initial Crack Size, Case 1, SF (5,5) Pair



Failure Assessment Diagram (FAD) Conservatism. The uncertainty of the FAD was taken from HSE (2000, [6]) in which the report performed the statistical analysis on the FAD, based on test data, as depicted by the scattered dots in Figure 8. The uncertainty is categorized into the fracture region, the fracture-plastic region and the plastic region. The elastic-plastic region shows relatively less scatter than the other two regions, and a Weibull distribution was fit to the scatter in this region. This Weibull distribution has a mean value of 0.47 and standard deviation 0.49, which is random variable *x11* in Table 1. Note that these are add-on conservative terms to the FAD in the radial distance going outward from the FAD.

Results

The graphical representations of the annual failure probability results are presented in Figure 10. A common trend is observed for all four cases, namely the annual failure probabilities are approximately constant after the safety factor pair is greater than (5,5). Note that the results presented in Figure 10 are annual failure probabilities for the final year of service (Year 30 for this study). Safety factor pair (1,1) produces high annual failure probability in all cases. Safety factor pair (2,2) shows high failure probability in Case 3, and low failure probability in Case 4. Safety factor pair (2,2) is likely to be located in the transition zone as depicted in Figure 3, due to the sensitivity observed for this pair. For other pairs no such sensitivities are observed.

Based on the findings of the investigated example problems, recommended safety factors are made as given in Table 2. Note that different applications have their own unique features, and these results should not be generalized without careful review of each specific application.

Conclusions

Structural reliability methods coupled with ECA analysis have been employed to calculate the failure probabilities for SCR welds when they contain various sizes of initial defects.

The results from the reliability analysis show that below a certain size of initial defect, the failure probabilities approach approximately a constant level corresponding to a cross section having no cracks. In other words, the reliability calculations have been reduced to failure being controlled by only plastic failure of an intact SCR cross section.

Figure 10 – Annual Failure Probability for Four Example Study Cases

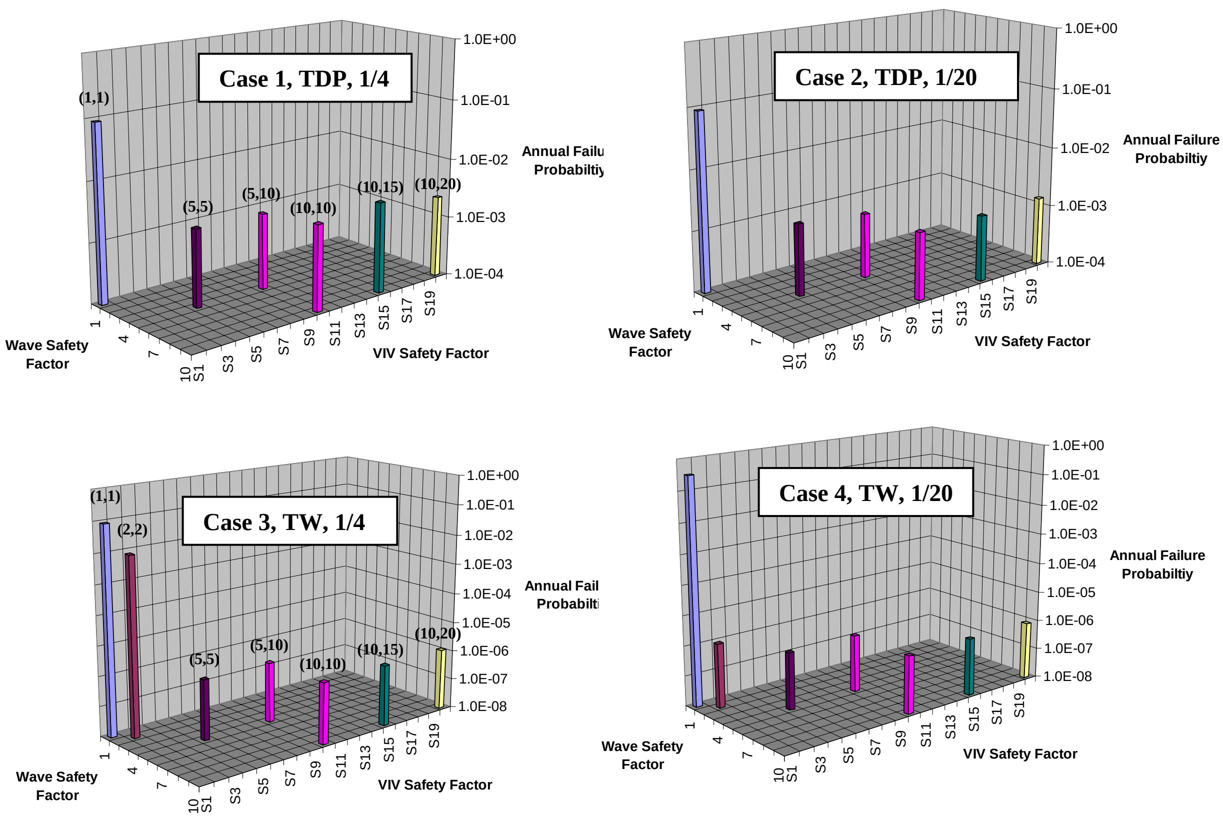


Table 2 – Results for Recommended Safety Factors (SF) for ECA Calculations for Example Cases

	SF for Wave-Induced Stress Ranges	SF for VIV-Induced Stress Ranges
Touch-Down Point Region	5	5
Top Weld Region	5	5

The results also indicate that for the cases studied, the safety factor pair (5,5) can be used in future ECA assessment on allowable initial crack sizes, without compromising the reliability of SCR performance. However, it is noted that each application has its own unique features, and these results should not be generalized without careful review of each specific case.

Acknowledgements

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